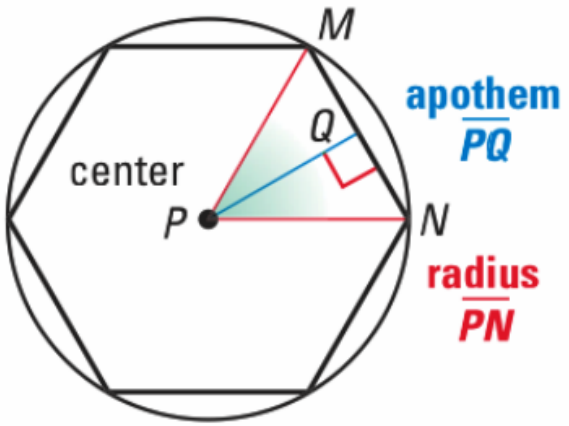
11.6 Areas of Regular Polygons

The diagram shows a regular polygon inscribed in a circle. The **center of the polygon** and the **radius of the polygon** are the center and the radius of its circumscribed circle.

The distance from the center to any side of the polygon is called the **apothem of the polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs.



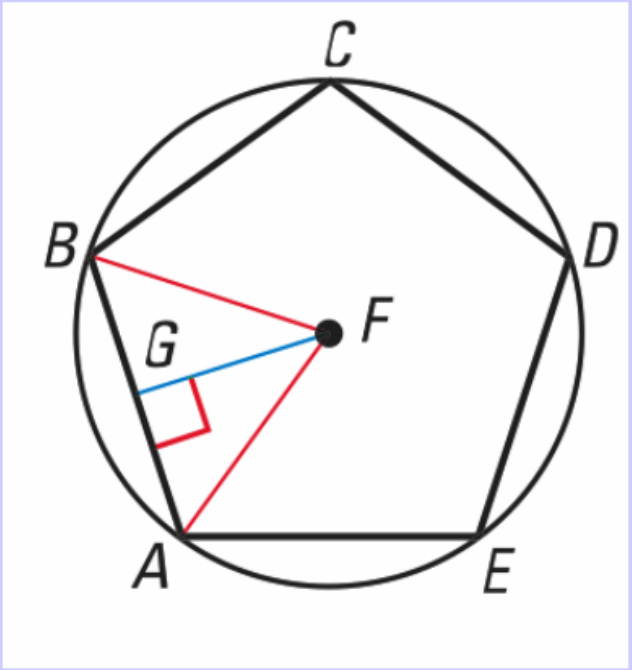
$\angle MPN$ is a central angle.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide 360° by the number of sides.

EXAMPLE 1 Find angle measures in a regular polygon

In the diagram, *ABCDE* is a regular pentagon inscribed in ⊙*F*. Find each angle measure.

a. $m\angle AFB$ **b.** $m\angle AFG$ **c.** $m\angle GAF$



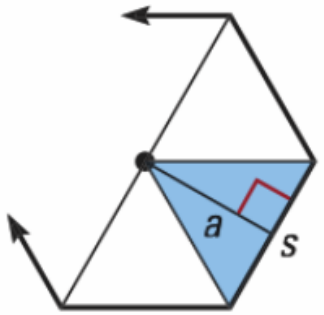
AREA OF AN *n*-GON You can find the area of any regular n-gon by dividing it into congruent triangles.

A = Area of one triangle • Number of triangles

$= \left(\frac{1}{2} \cdot s \cdot a\right) \cdot n$ **Base of triangle is *s* and height of triangle is *a*. Number of triangles is *n*.**

$= \frac{1}{2} \cdot a \cdot (n \cdot s)$ **Commutative and Associative Properties of Equality**

$= \frac{1}{2} a \cdot P$ **There are *n* congruent sides of length *s*, so perimeter *P* is *n* • *s*.**

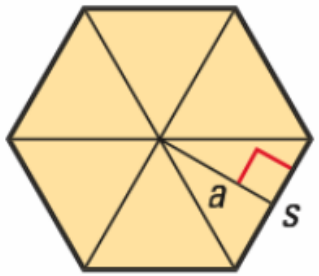


THEOREM

For Your Notebook

THEOREM 11.11 Area of a Regular Polygon

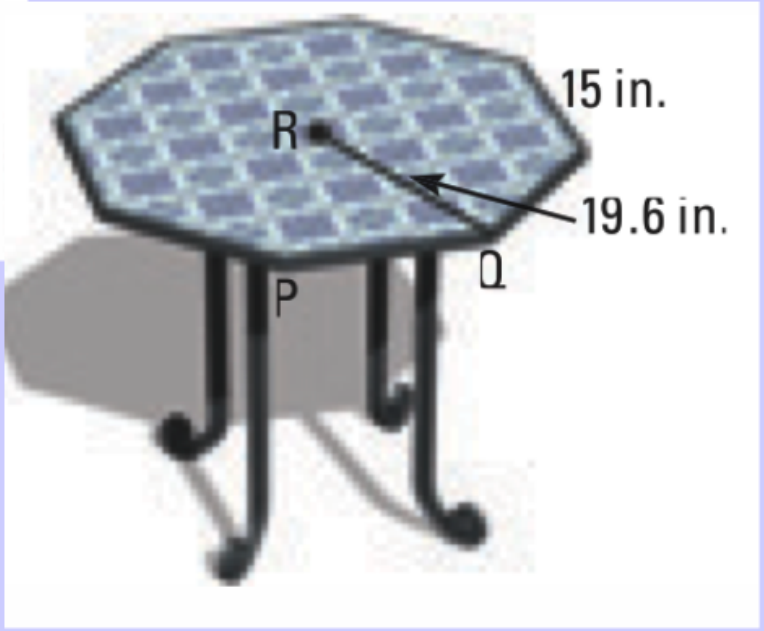
The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$.

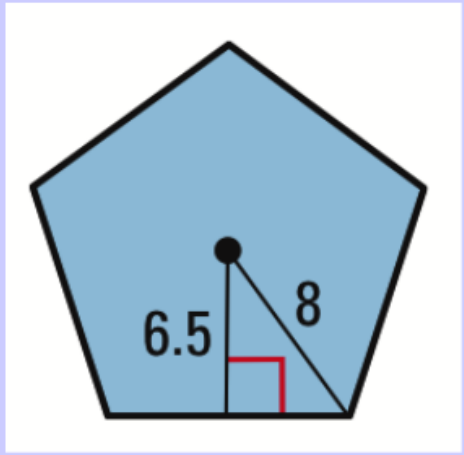


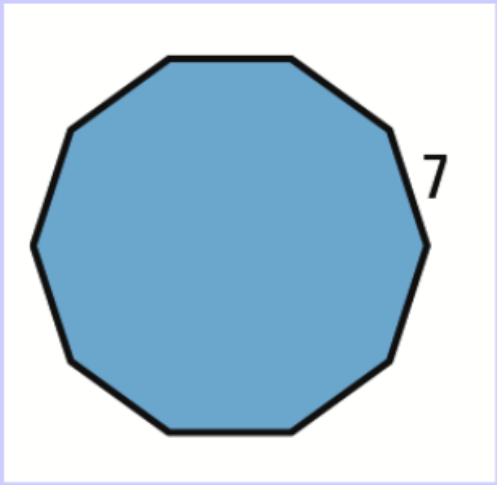
EXAMPLE 2 Find the area of a regular polygon

Find the perimeter and the area of the regular polygon.

DECORATING You are decorating the top of a table by covering it with small ceramic tiles. The table top is a regular octagon with 15 inch sides and a radius of about 19.6 inches. What is the area you are covering?




6.5
8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CONCEPT SUMMARY *For Your Notebook*

Finding Lengths in a Regular *n*-gon

To find the area of a regular *n*-gon with radius *r*, you may need to first find the apothem ***a*** or the side length ***s***.

You can use . . .	**. . . when you know *n* and . . .**	**. . . as in . . .**
Pythagorean Theorem: $\left(\frac{1}{2}s\right)^2 + a^2 = r^2$	Two measures: *r* and ***a***, or *r* and ***s***	Example 2 and Guided Practice Ex. 3.
Special Right Triangles	Any one measure: *r* or ***a*** or ***s*** **And** the value of *n* is 3, 4, or 6	Guided Practice Ex. 5.
Trigonometry	Any one measure: *r* or ***a*** or ***s***	Example 3 and Guided Practice Exs. 4 and 5.

Assignment:

Day 1: 11.6 ws (odds)

Day 2: 11.6 ws (evens)

LESSON 11.6

Practice

For use with pages 762–769

Find the measure of a central angle of a regular polygon with the given number of sides. Round answers to the nearest tenth of a degree, if necessary.

1. 20 sides

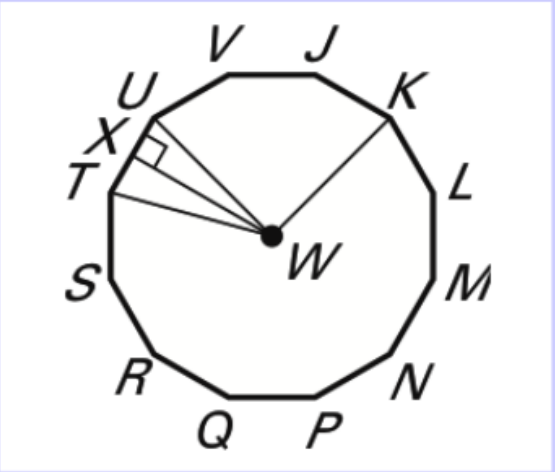
2. 36 sides

3. 120 sides

4. 23 sides

Find the given angle measure for the regular dodecagon shown.

5. $m\angle TWU$ **6.** $m\angle TWX$



7. $m\angle XUW$ **8.** $m\angle TWK$

9. $m\angle UWK$ **10.** $m\angle XWK$

11. Multiple Choice Which expression gives the apothem for a regular nonagon with side length 10.5?

A. $a = \frac{5.25}{\tan 40°}$

B. $a = \frac{10.5}{\tan 20°}$

C. $a = \frac{5.25}{\tan 20°}$

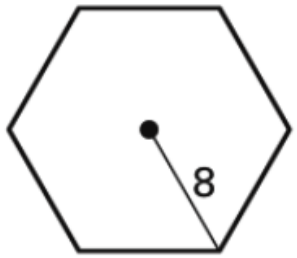
D. $a = 5.25 \cdot \tan 20°$

12. A regular hexagon has a diameter 22 inches. What is the length of its apothem? Round your answer to the nearest tenth.

13. A regular octagon has a diameter 8.5 feet. What is the length of its apothem? Round your answer to the nearest tenth.

Find the perimeter and area of the regular polygon. Round answers to the nearest tenth, if necessary.

14.



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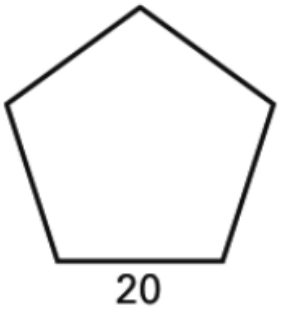
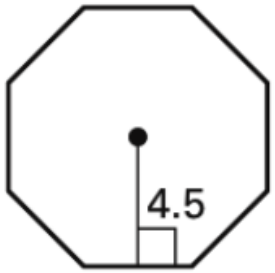


16.

17.



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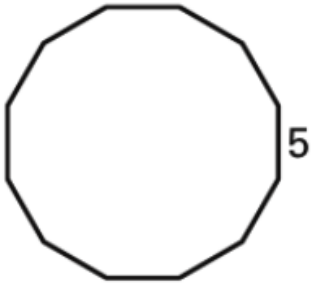

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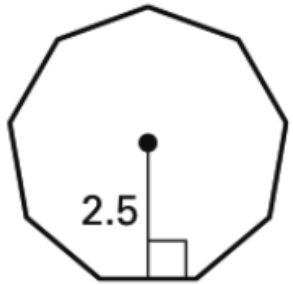


20. What is the area of a regular 18-gon with a side length of 8 meters? Round your answer to the nearest tenth, if necessary.

21. What is the area of a regular 24-gon with a side length of 10 inches? Round your answer to the nearest tenth, if necessary.

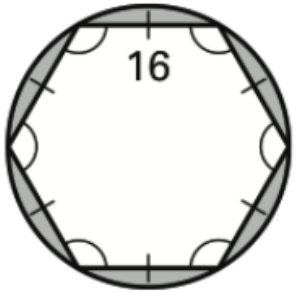
22. What is the area of a regular 30-gon with a radius of 20 feet? Round your answer to the nearest tenth, if necessary.

23. Find the area of a regular pentagon inscribed in a circle whose equation is given by $(x-4)^2+(y-6)^2=16$.

24. Find the area of a regular octagon inscribed in a circle whose equation is given by $(x - 2)^2 + (y + 3)^2 = 25$.

Find the area of the shaded region. Round answers to the nearest tenth, if necessary.

25.



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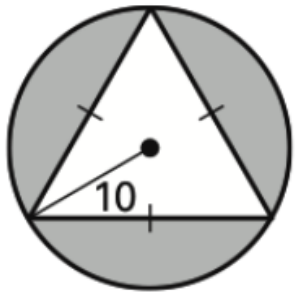

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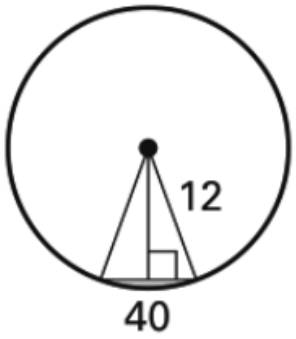

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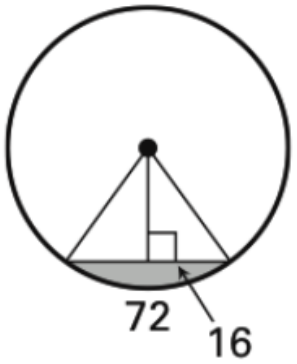
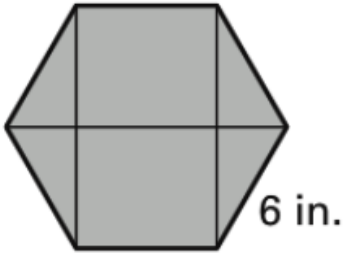


In Exercises 29 and 30, use the following information.

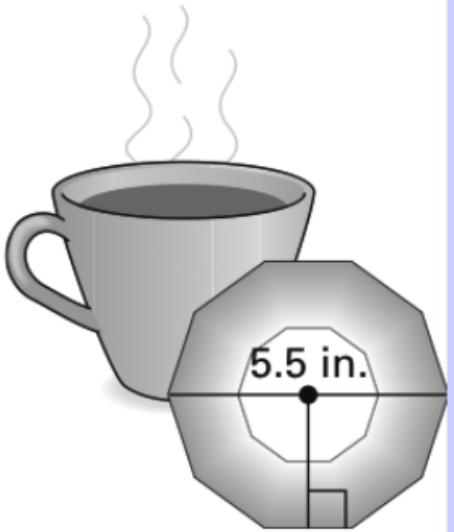
Tiles You are tiling the floor of a hallway with tiles that are regular hexagons as shown.



29. What is the area of each tile?

30. The hallway has a width of 5 feet and a length of 12 feet. At least how many tiles will you need?

31. A cup saucer is shaped like a regular decagon with a diameter of 5.5 inches as shown.



a. What is the length of the apothem of the saucer? Round your answer to the nearest tenth.

b. What is the perimeter and area of the saucer? Round your answers to the nearest tenth.