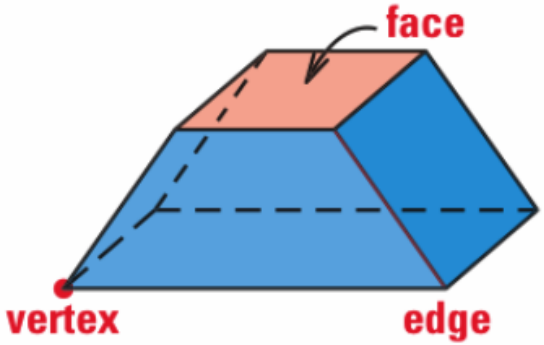
12.1 Explore Solids

A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.

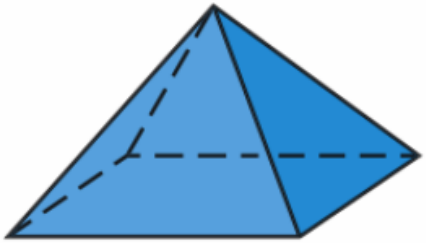


KEY CONCEPT — For Your Notebook

Types of Solids

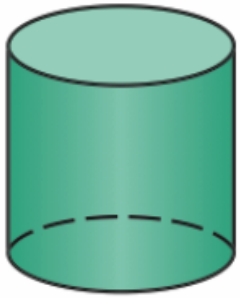
Polyhedra

Prism

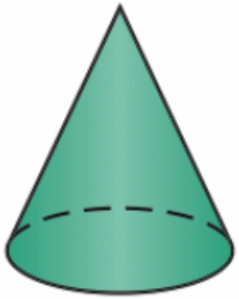


Pyramid

Not Polyhedra



Cylinder

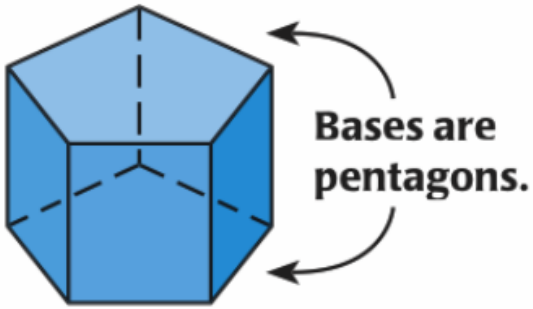


Cone

Sphere

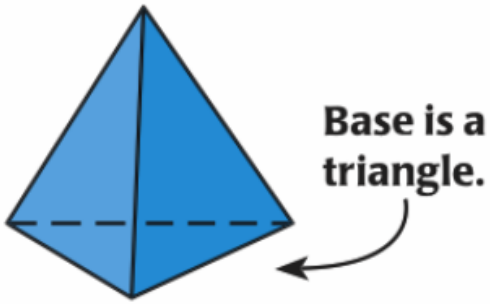
CLASSIFYING SOLIDS Of the five solids above, the prism and the pyramid are polyhedra. To name a prism or a pyramid, use the shape of the *base*.

Pentagonal prism



The two **bases** of a prism are congruent polygons in parallel planes.

Triangular pyramid

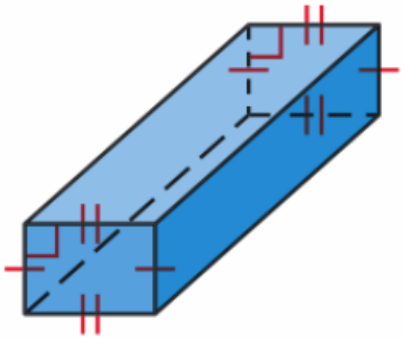


The **base** of a pyramid is a polygon.

EXAMPLE 1 Identify and name polyhedra

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

a.

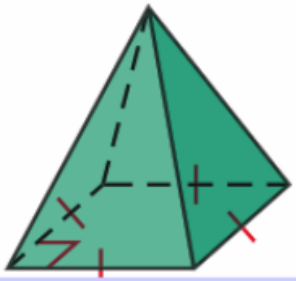


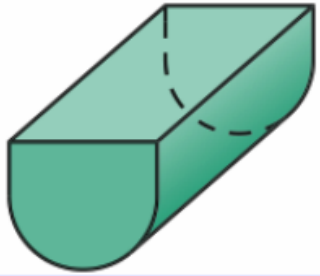
b.

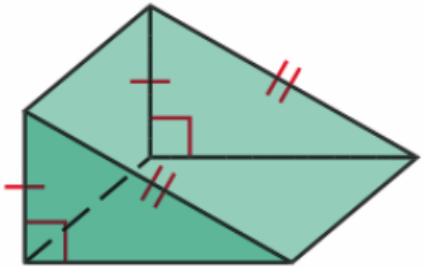
c.

GUIDED PRACTICE for Example 1

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

1. 

2. 

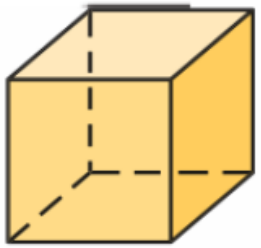
3. 

EULER'S THEOREM Notice in Example 1 that the sum of the number of faces and vertices of the polyhedra is two more than the number of edges. This suggests the following theorem, proved by the Swiss mathematician Leonhard Euler (pronounced "oi'-ler"), who lived from 1707 to 1783.

THEOREM *For Your Notebook*

THEOREM 12.1 Euler's Theorem

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

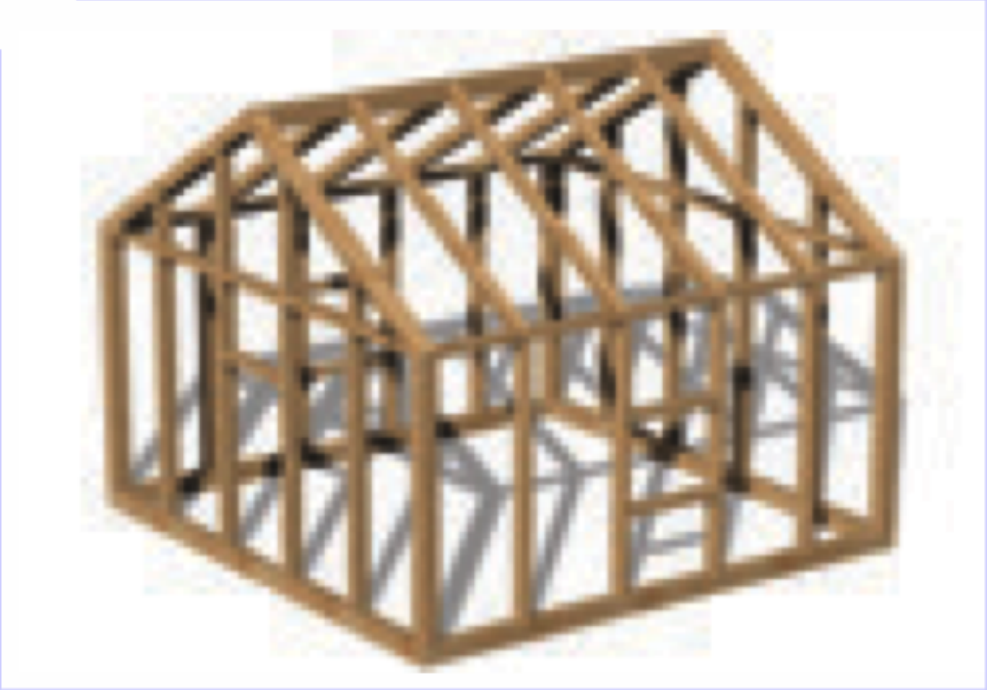


$F = 6, V = 8, E = 12$

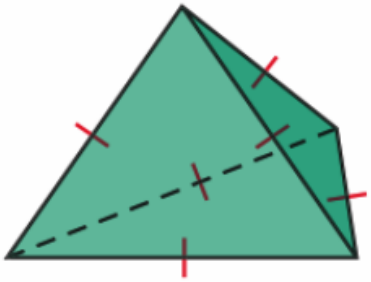
$6 + 8 = 12 + 2$

EXAMPLE 2 Use Euler's Theorem in a real-world situation

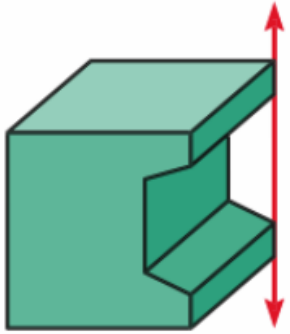
HOUSE CONSTRUCTION Find the number of edges on the frame of the house.



REGULAR POLYHEDRA A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is nonconvex, or *concave*.

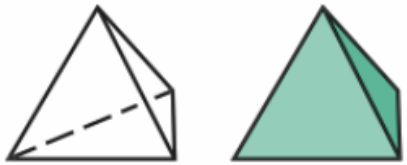


regular, convex

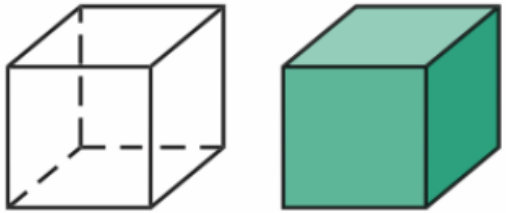


nonregular, concave

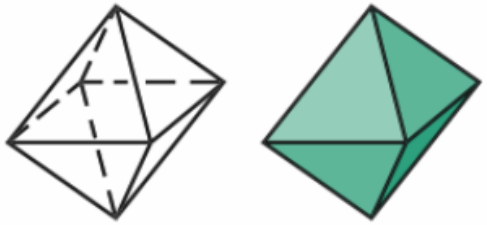
There are five regular polyhedra, called **Platonic solids** after the Greek philosopher Plato (c. 427 B.C.–347 B.C.). The five Platonic solids are shown.



Regular tetrahedron
4 faces



Cube
6 faces



Regular octahedron
8 faces

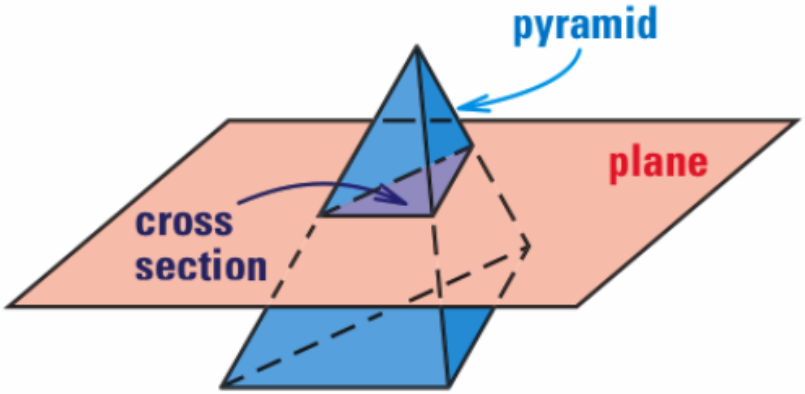
Regular dodecahedron
12 faces

Regular icosahedron
20 faces

EXAMPLE 3 Use Euler's Theorem with Platonic solids

Find the number of faces, vertices, and edges of the regular octahedron. Check your answer using Euler's Theorem.

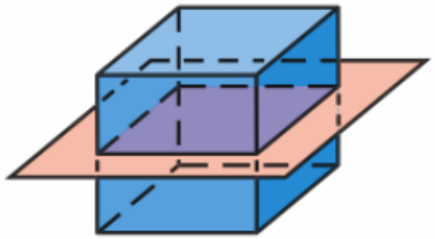
CROSS SECTIONS Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, the diagram shows that an intersection of a plane and a triangular pyramid is a triangle.



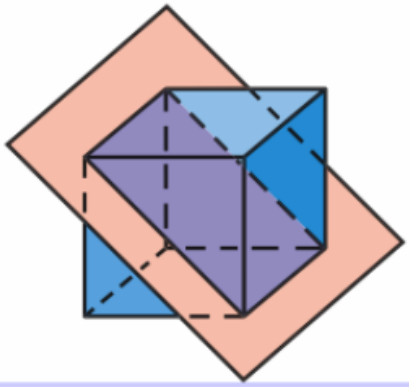
EXAMPLE 4 Describe cross sections

Describe the shape formed by the intersection of the plane and the cube.

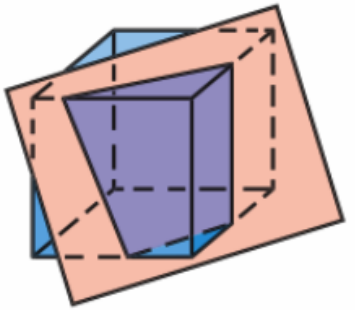
a.



b.

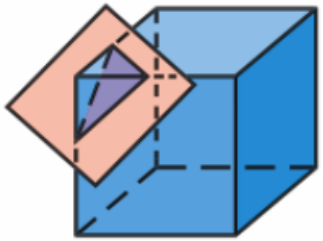


c.

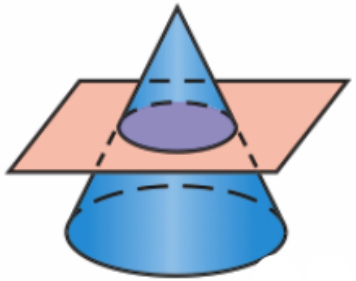


Describe **the shape formed by the intersection of the plane and the solid.**

5.



6.



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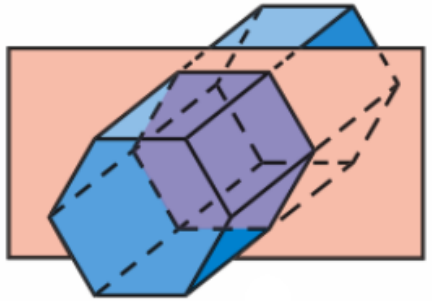

Assignment:

12.1 ws