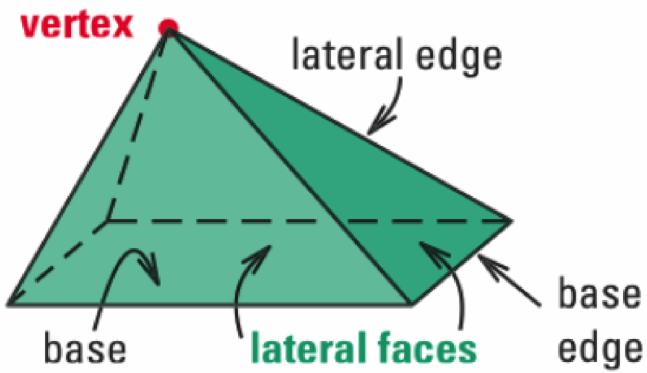
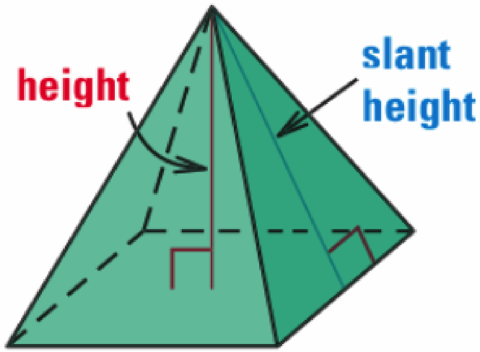
12.3 Surface Area of Pyramids and Cones

A **pyramid** is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex, called the **vertex of the pyramid**. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The height of the pyramid is the perpendicular distance between the base and the vertex.

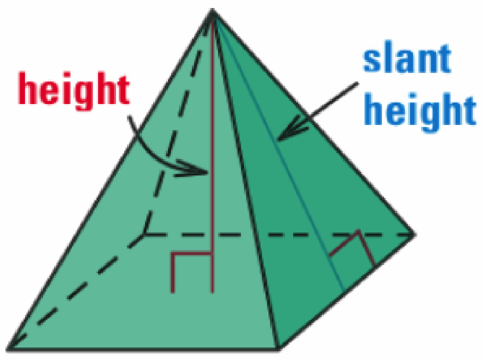


Pyramid



Regular pyramid

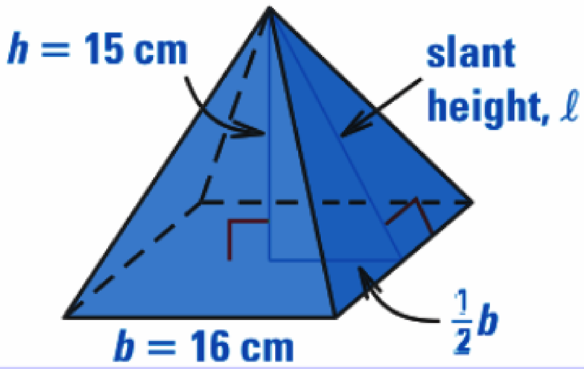
A **regular pyramid** has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base. The lateral faces of a regular pyramid are congruent isosceles triangles. The **slant height** of a regular pyramid is the height of a lateral face of the regular pyramid. A nonregular pyramid does not have a slant height.



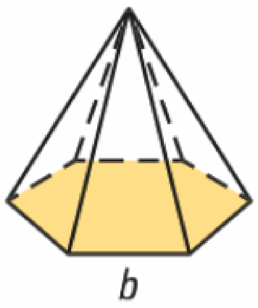
Regular pyramid

EXAMPLE 1 Find the area of a lateral face of a pyramid

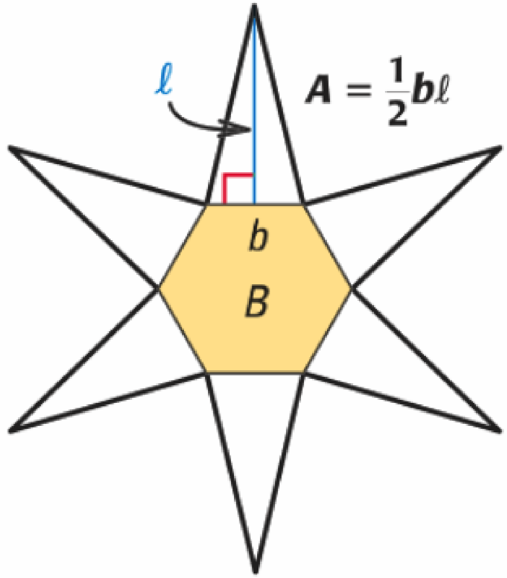
A regular square pyramid has a height of 15 centimeters and a base edge length of 16 centimeters. Find the area of each lateral face of the pyramid.



SURFACE AREA A regular hexagonal pyramid and its net are shown at the right. Let b represent the length of a base edge, and let ℓ represent the slant height of the pyramid.



The area of each lateral face is $\frac{1}{2}b\ell$ and the perimeter of the base is $P = 6b$. So, the surface area S is as follows.



$S =$ (Area of base) $+ 6$(Area of lateral face)

$S = B + 6\left(\frac{1}{2}b\ell\right)$ **Substitute.**

$S = B + \frac{1}{2}(6b)\ell$ **Rewrite $6\left(\frac{1}{2}b\ell\right)$ as $\frac{1}{2}(6b)\ell$.**

$S = B + \frac{1}{2}P\ell$ **Substitute P for $6b$.**

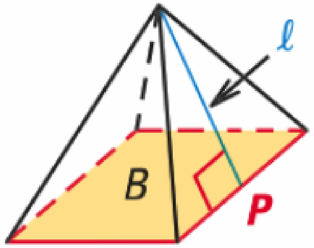
THEOREM *For Your Notebook*

THEOREM 12.4 Surface Area of a Regular Pyramid

The surface area S of a regular pyramid is

$$S = B + \frac{1}{2}P\ell,$$

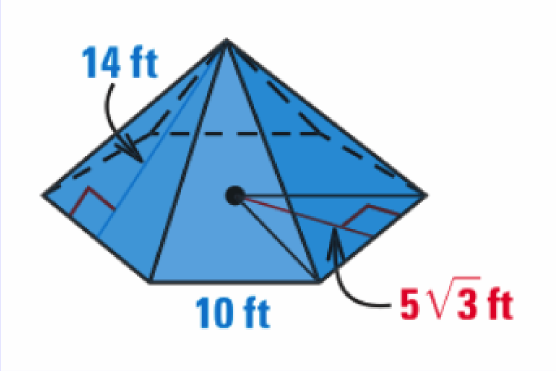
where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



$S = B + \frac{1}{2}P\ell$

EXAMPLE 2 Find the surface area of a pyramid

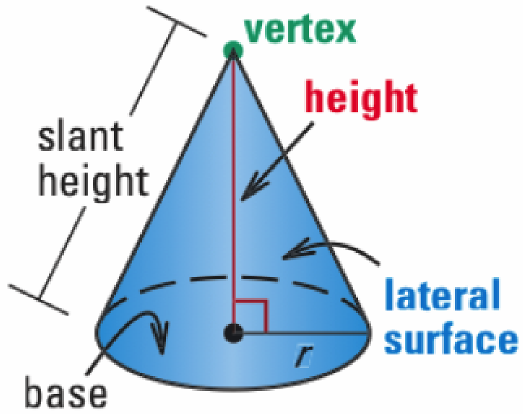
Find the surface area of the regular hexagonal pyramid.



CONES A **cone** has a circular base and a **vertex** that is not in the same plane as the base. The radius of the base is the *radius* of the cone. The height is the perpendicular distance between the vertex and the base.

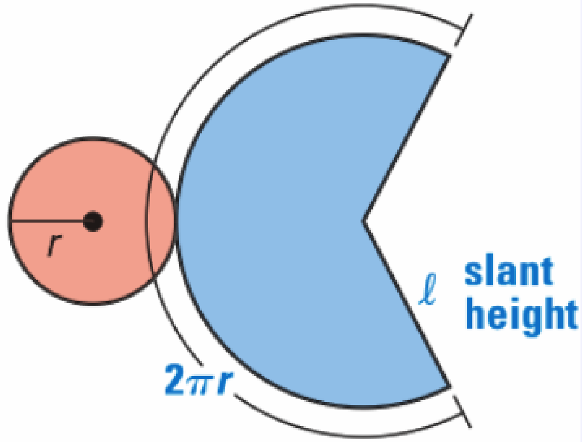
In a **right cone**, the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge.



Right cone

SURFACE AREA When you cut along the slant height and base edge and lay a right cone flat, you get the net shown at the right.



The circular base has an area of πr^2 and the lateral surface is the sector of a circle. You can use a proportion to find the area of the sector, as shown below.

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}}$$ **Set up proportion.**

$$\frac{\text{Area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell}$$ **Substitute.**

$$\text{Area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell}$$ **Multiply each side by $\pi \ell^2$.**

$$\text{Area of sector} = \pi r \ell$$ **Simplify.**

The surface area of a cone is the sum of the base area, πr^2, and the lateral area, $\pi r \ell$. Notice that the quantity $\pi r \ell$ can be written as $\frac{1}{2}(2\pi r)\ell$, or $\frac{1}{2}C\ell$.

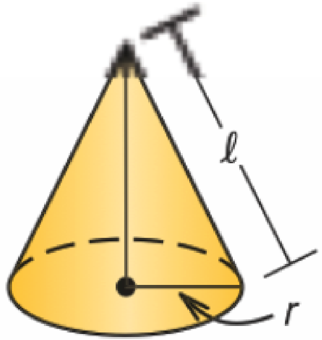
THEOREM *For Your Notebook*

THEOREM 12.5 Surface Area of a Right Cone

The surface area S of a right cone is

$$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell,$$

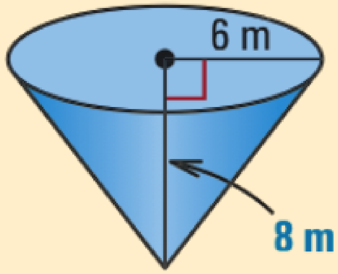
where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.



$$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$$

EXAMPLE 3 Standardized Test Practice

What is the surface area of the right cone?



(A) $72\pi\ \text{m}^2$ (B) $96\pi\ \text{m}^2$

(C) $132\pi\ \text{m}^2$ (D) $136\pi\ \text{m}^2$

Assignment

Day 1:

12.3 ws

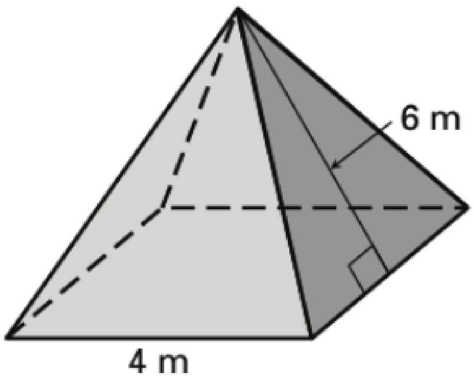
LESSON 12.3

Practice

For use with pages 810–817

Find the area of each lateral face of the regular pyramid. Round your answer to two decimal places.

1.



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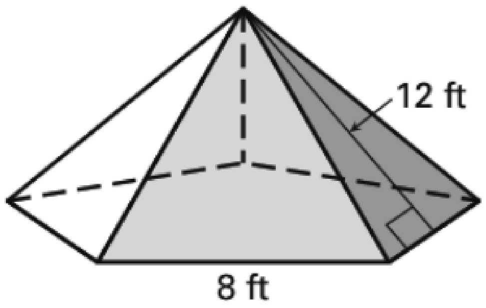

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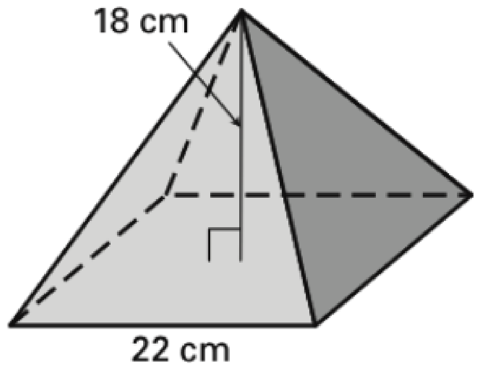
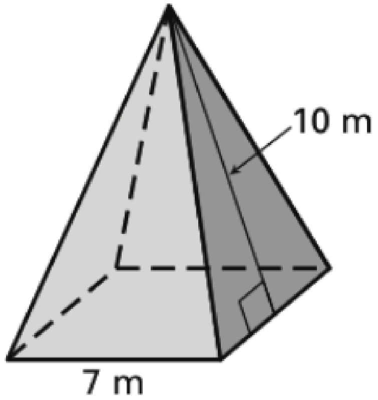


Find the surface area of the regular pyramid. Round your answer to two decimal places.

4.



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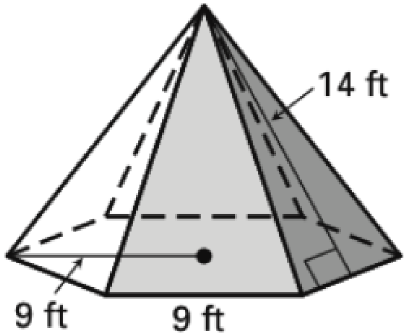

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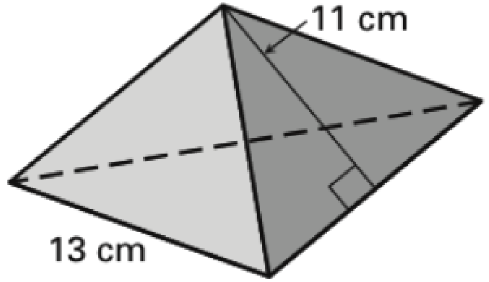
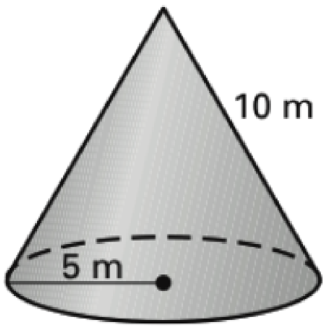


Find the lateral area of the right cone. Round your answer to two decimal places.

7.



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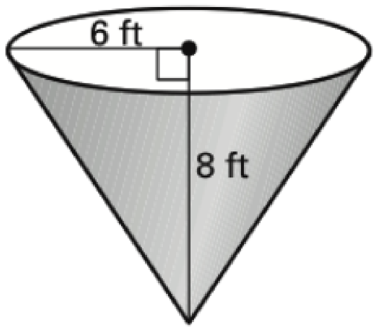

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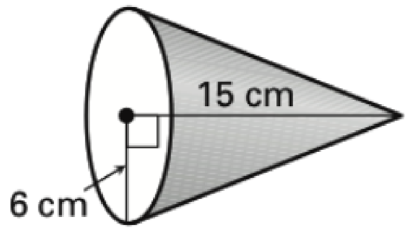


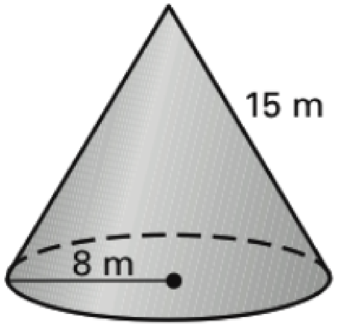
LESSON 12.3

Practice *continued*

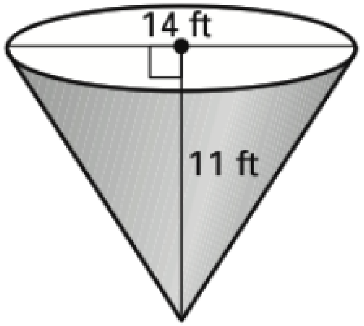
For use with pages 810–817

Find the surface area of the right cone. Round your answer to two decimal places.

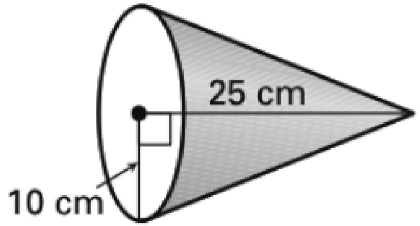
10.



11.



12.



13. **Multiple Choice** The surface area of a regular pyramid with a square base is 1536 square meters. The base edge length is 24 meters and the slant height is 20 meters. What is the height of the pyramid?

A. 8 meters **B.** 12 meters **C.** 16 meters **D.** 20 meters

Sketch the described solid and find its surface area. Round your answer to two decimal places.

14. A regular pyramid has a slant height of 12 inches. Its base is a square with a base edge length of 18 inches.

15. A regular pyramid has a height of 10 inches. Its base is an equilateral triangle with a base edge length of 12 inches.

16. A right cone has a radius of 3 feet and a height of 9 feet.

17. A right cone has a diameter of 12 meters and a slant height of 9 meters.

Assignment Day 2:

p. 814 (3-15, 22-24, 35-39 all)