2.6 Prove Statements about Segments and Angles

Goal • Write proofs using geometric theorems.

Proof-Logical argument that shows a statement is true.

Two-column proof-has numbered statements and corresponding reasons that show an argument in a logical order

*In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made.

*Each reason in the right-hand column is the explanation for the corresponding statement.

Statements	Reasons

EXAMPLE 1 Write a two-column proof

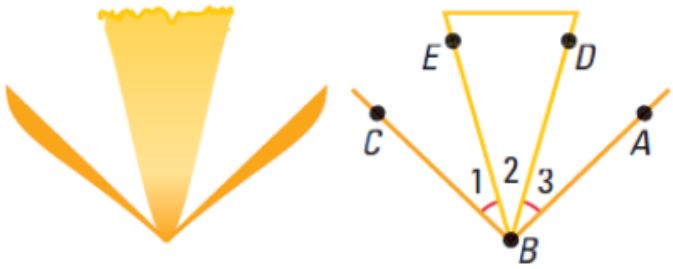
WRITE PROOFS

Writing a two-column proof is a formal way of organizing your reasons to show a statement is true.

Write a two-column proof for the situation in Example 4 on page 107.

GIVEN ▶ $m\angle 1 = m\angle 3$

PROVE ▶ $m\angle EBA = m\angle DBC$



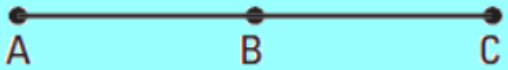
STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	**1.** Given
2. $m\angle EBA = m\angle 3 + m\angle 2$	**2.** Angle Addition Postulate
3. $m\angle EBA = m\angle 1 + m\angle 2$	**3.** Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle DBC$	**4.** Angle Addition Postulate
5. $m\angle EBA = m\angle DBC$	**5.** Transitive Property of Equality

GUIDED PRACTICE for Example 1

1. Four steps of a proof are shown. Give the reasons for the last two steps.

GIVEN ▶ $AC = AB + AB$

PROVE ▶ $AB = BC$



STATEMENTS	REASONS
1. $AC = AB + AB$	**1.** Given
2. $AB + BC = AC$	**2.** Segment Addition Postulate
3. $AB + AB = AB + BC$	**3.** ?
4. $AB = BC$	**4.** ?

Reasons in the proof can include:

*Definitions

*Properties

*Postulates

*Theorems

Theorem- a statement that can be proven

*Once you have proven a theorem, you can use the theorem as a reason in other proofs.

TAKE NOTES

Be sure to copy all new theorems in your notebook. Notice that the theorem box tells you where to find the proof(s).

THEOREMS

For Your Notebook

THEOREM 2.1 Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Proofs: p. 137; Ex. 5, p. 121; Ex. 26, p. 118

THEOREM 2.2 Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proofs: Ex. 25, p. 118; Concept Summary, p. 114; Ex. 21, p. 137

EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.

a. If $\angle R \cong \angle T$ and $\angle T \cong \angle P$, then $\angle R \cong \angle P$.

b. If $\overline{NK} \cong \overline{BD}$, then $\overline{BD} \cong \overline{NK}$.

GUIDED PRACTICE for Example 2

Name the property illustrated by the statement.

2. $\overline{CD} \cong \overline{CD}$

3. If $\angle Q \cong \angle V$, then $\angle V \cong \angle Q$.

*In this lesson, most of the proofs involve showing that congruence and equality are equivalent. You may find that what you are asked to prove seems to be obviously true. It is important to practice writing these proofs so that you will be prepared to write more complicated proofs in later chapters.

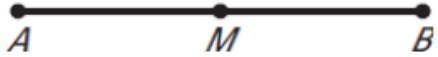
EXAMPLE 3 Use properties of equality

Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one half of AB.

WRITE PROOFS

Before writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the GIVEN and PROVE statements.

GIVEN ▶ M is the midpoint of $\overline{AB}$.



PROVE ▶ **a.** $AB = 2 \cdot AM$

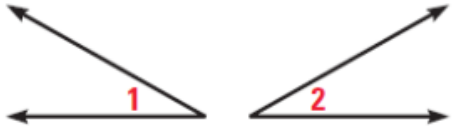
b. $AM = \frac{1}{2}AB$

STATEMENTS	REASONS
1. M is the midpoint of $\overline{AB}$.	**1.** Given
2. $\overline{AM} \cong \overline{MB}$	**2.** Definition of midpoint
3. $AM = MB$	**3.** Definition of congruent segments
4. $AM + MB = AB$	**4.** Segment Addition Postulate
5. $AM + AM = AB$	**5.** Substitution Property of Equality
a. 6. $2AM = AB$	**6.** Distributive Property
b. 7. $AM = \frac{1}{2}AB$	**7.** Division Property of Equality

CONCEPT SUMMARY — For Your Notebook

Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs.

Proof of the Symmetric Property of Angle Congruence

GIVEN ▶ $\angle 1 \cong \angle 2$

PROVE ▶ $\angle 2 \cong \angle 1$

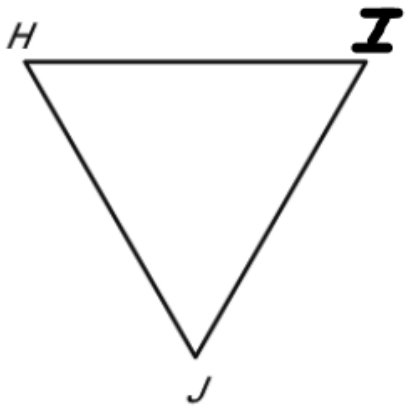
STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. **Given**
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Statements based on facts that you know or on conclusions from deductive reasoning →

← Definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.



1. GIVEN: $HI = 9, IJ = 9, \overline{IJ} \cong \overline{JH}$

PROVE: $\overline{HI} \cong \overline{JH}$

Statements	Reasons
1. $HI = 9$	**1.** ?
2. $IJ = 9$	**2.** ?
3. $HI = IJ$	**3.** ?
4. ?	**4.** Definition of congruent segments
5. $\overline{IJ} \cong \overline{JH}$	**5.** ?
6. $\overline{HI} \cong \overline{JH}$	**6.** ?

3. **GIVEN:** $AL = SK$

PROVE: $AS = LK$

Statements	Reasons
1. $AL = SK$	**1.** ?
2. $LS = LS$	**2.** ?
3. $AL + LS = SK + LS$	**3.** ?
4. $AL + LS = AS$	**4.** ?
5. $SK + LS = LK$	**5.** ?
6. $AS = LK$	**6.** ?

Assignment:

p. 116 (1-13, 16-18, 21-24, 31-36 all)