

2.7

Prove Angle Pair Relationships

Goal • Use properties of special pairs of angles.

*Sometimes, a new theorem describes a relationship that is useful in writing proofs. For example, using the *Right Angles Congruence Theorem* is a step you may need to include in a proof involving right angles.

THEOREM

For Your Notebook

THEOREM 2.3 Right Angles Congruence Theorem

All right angles are congruent.

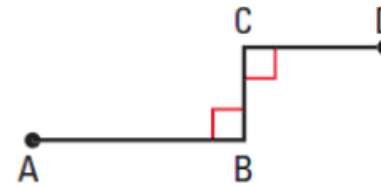
Proof: below

EXAMPLE 1 Use right angle congruence

Write a proof.

GIVEN ▶ $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$

PROVE ▶ $\angle B \cong \angle C$



AVOID ERRORS

The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show the angles are right angles.

STATEMENTS

- 1.
- 2.
- 3.

REASONS

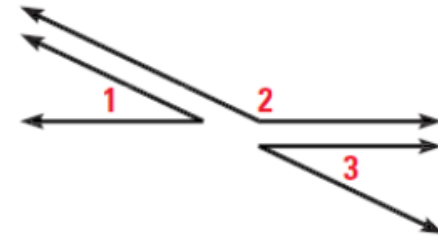
- 1.
- 2.
- 3.

THEOREMS*For Your Notebook***THEOREM 2.4 Congruent Supplements Theorem**

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

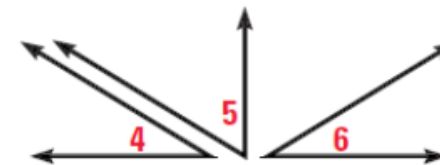
Proof: Example 2, below; Ex. 36, p. 129

**THEOREM 2.5 Congruent Complements Theorem**

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

Proof: Ex. 37, p. 129; Ex. 41, p. 130

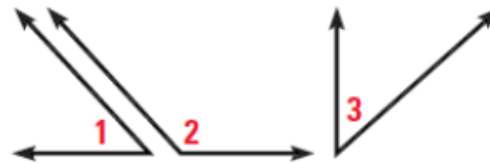


EXAMPLE 2 Prove a case of Congruent Supplements Theorem

Prove that two angles supplementary to the same angle are congruent.

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements.
 $\angle 3$ and $\angle 2$ are supplements.

PROVE ▶ $\angle 1 \cong \angle 3$



STATEMENTS

REASONS

1.

1.

2.

2.

3.

3.

4.

4.

5.

5.

Day 1 Assignment:

p. 127 (3-5, 7, 36-39, 49-53)

2.7 Day 2:

*When two lines intersect, pairs of vertical angles and linear pairs are formed. The relationship that you used in Lesson 1.5 for linear pairs is formally stated below as the *Linear Pair Postulate*. This postulate is used for the proof of the *Congruence Theorem*.

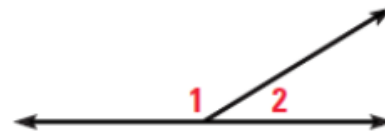
POSTULATE

For Your Notebook

POSTULATE 12 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

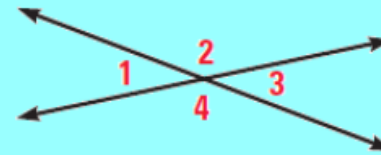
$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180^\circ$.



THEOREM*For Your Notebook***THEOREM 2.6 Vertical Angles Congruence Theorem**

Vertical angles are congruent.

Proof: Example 3, below



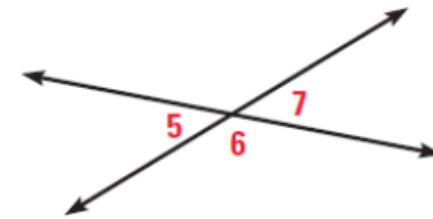
$$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$$

EXAMPLE 3 Prove the Vertical Angles Congruence Theorem

Prove vertical angles are congruent.

GIVEN ► $\angle 5$ and $\angle 7$ are vertical angles.

PROVE ► $\angle 5 \cong \angle 7$



USE A DIAGRAM

You can use information labeled in a diagram in your proof.

STATEMENTS

1. $\angle 5$ and $\angle 7$ are vertical angles.
2. $\angle 5$ and $\angle 6$ are a linear pair.
 $\angle 6$ and $\angle 7$ are a linear pair.
3. $\angle 5$ and $\angle 6$ are supplementary.
 $\angle 6$ and $\angle 7$ are supplementary.
4. $\angle 5 \cong \angle 7$

REASONS

1. Given
2. Definition of linear pair, as shown in the diagram
3. Linear Pair Postulate
4. Congruent Supplements Theorem



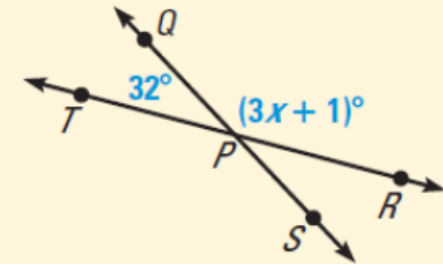
EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES

Look for angle pair relationships in the diagram. The angles in the diagram are supplementary, not complementary or congruent, so eliminate choices A and C.

Which equation can be used to find x ?

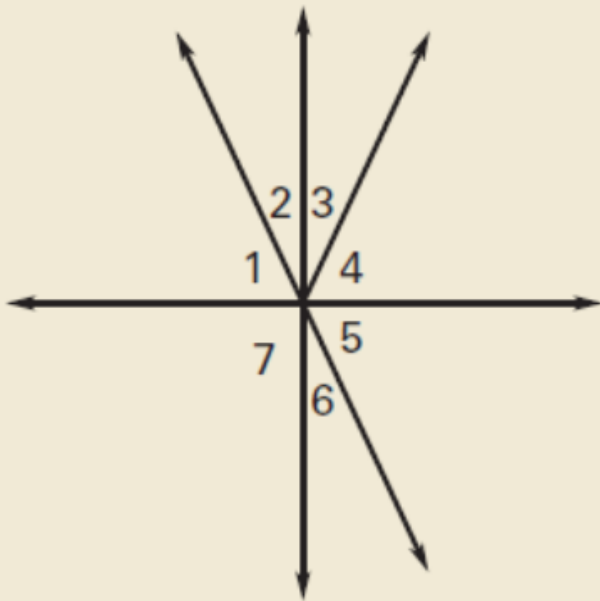
- (A) $32 + (3x + 1) = 90$
- (B) $32 + (3x + 1) = 180$
- (C) $32 = 3x + 1$
- (D) $3x + 1 = 212$



Solution

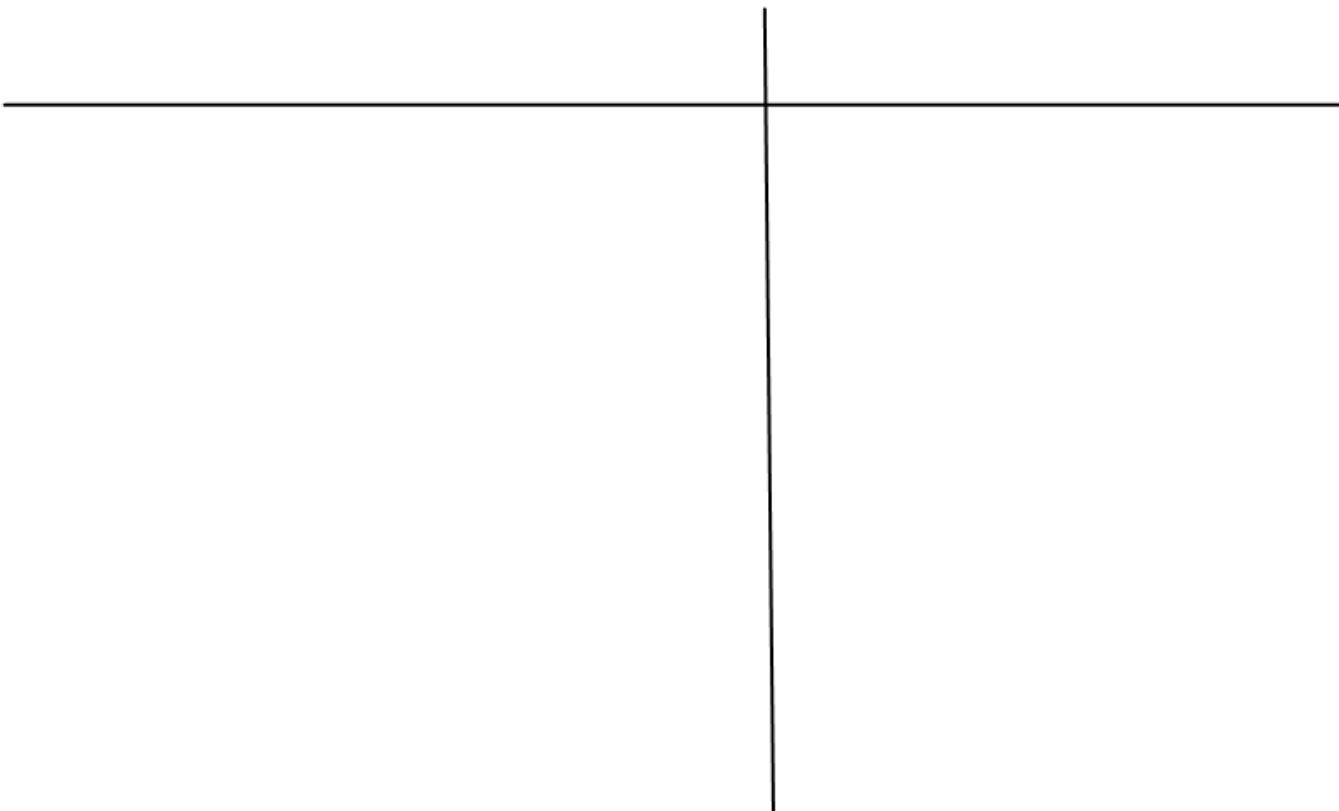


Write a proof.



Given: $\angle 2 \cong \angle 3$

Prove: $\angle 3 \cong \angle 6$



Day 2 Assignment:

p. 128 (8-14, 16-29, 31-34, 42-46 all)