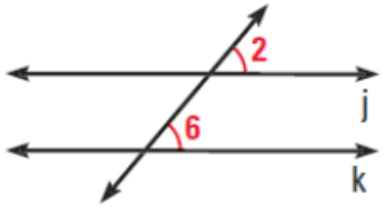
3.3 Prove Lines are Parallel

Goal • Use angle relationships to prove that lines are parallel.

POSTULATE *For Your Notebook*

POSTULATE 16 Corresponding Angles Converse

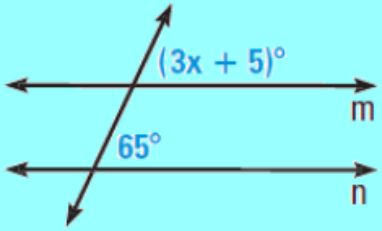
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



$j \parallel k$

EXAMPLE 1 Apply the Corresponding Angles Converse

ALGEBRA Find the value of x that makes $m \parallel n$.



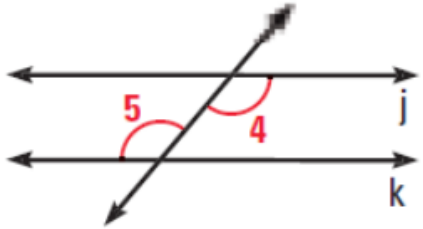
THEOREMS

For Your Notebook

THEOREM 3.4 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Proof: Example 3, p. 163

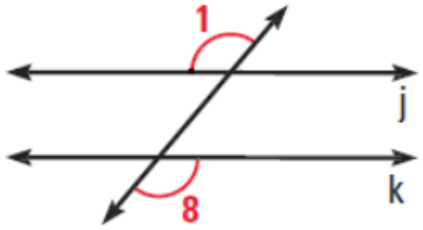


$j \parallel k$

THEOREM 3.5 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

Proof: Ex. 36, p. 168

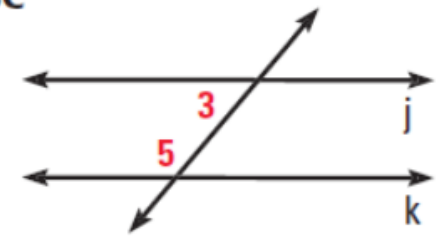


$j \parallel k$

THEOREM 3.6 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

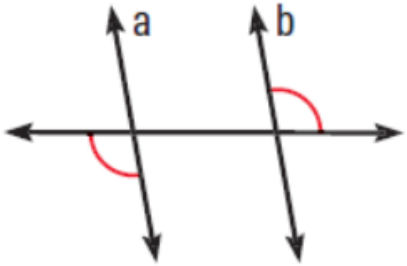
Proof: Ex. 37, p. 168



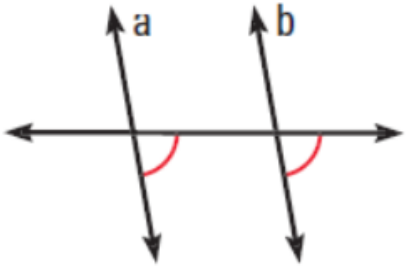
If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

Can you prove that lines *a* and *b* are parallel? *Explain* why or why not.

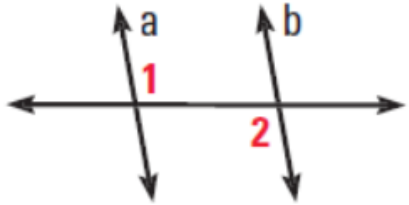
3.



4.



5. $m\angle 1 + m\angle 2 = 180°$



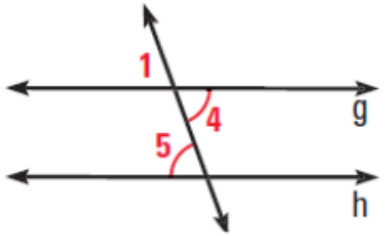
EXAMPLE 3 Prove the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Solution

GIVEN ▶ $\angle 4 \cong \angle 5$

PROVE ▶ $g \parallel h$



AVOID ERRORS

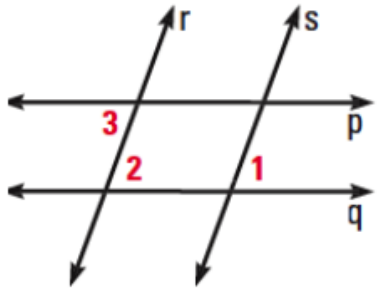
Before you write a proof, identify the GIVEN and PROVE statements for the situation described or for any diagram you draw.

*__paragraph proof-__ a proof that is written as a paragraph

the statements and reasons are written in sentences, using words to explain the logical flow of the argument

EXAMPLE 4 Write a paragraph proof

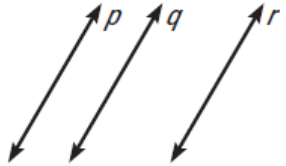
In the figure, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.



THEOREM *For Your Notebook*

THEOREM 3.7 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

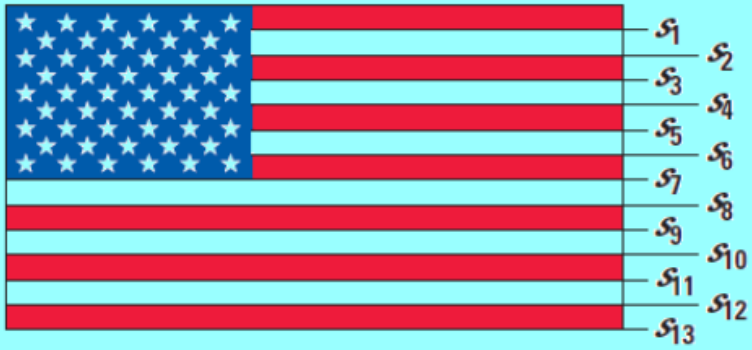


If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Proofs: Ex. 38, p. 168; Ex. 38, p. 177

EXAMPLE 5 Use the Transitive Property of Parallel Lines

U.S. FLAG The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.



Solution

USE SUBSCRIPTS
When you name several similar items, you can use one variable with subscripts to keep track of the items.

The stripes from top to bottom can be named $s_1, s_2, s_3, \ldots, s_{13}$. Each stripe is parallel to the one below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$. So, the top stripe is parallel to the bottom stripe.

Day 1 Assignment:

3.3 ws

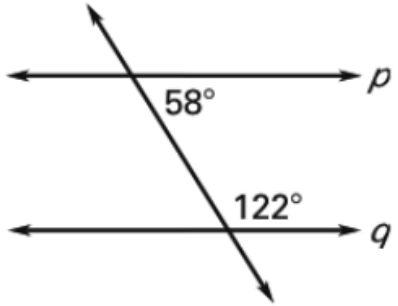
LESSON 3.3

Practice

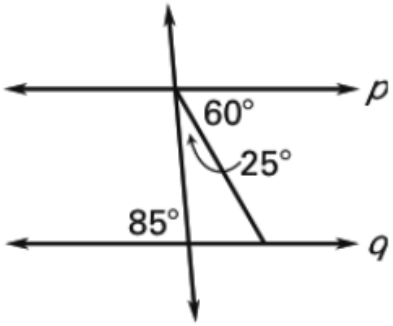
For use with pages 161–169

Is it possible to prove that lines *p* and *q* are parallel? If so, state the postulate or theorem you would use.

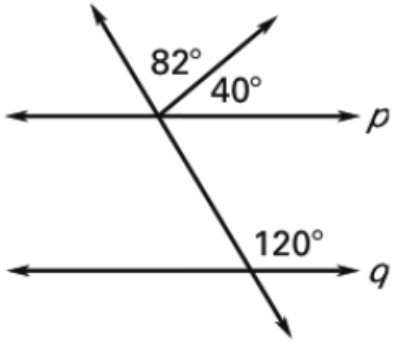
1.



2.

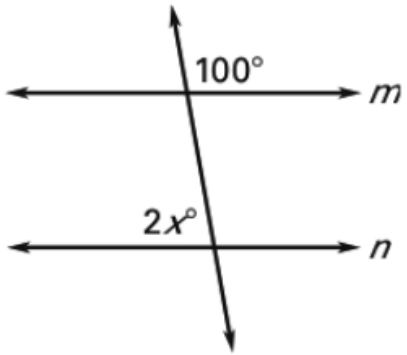


3.

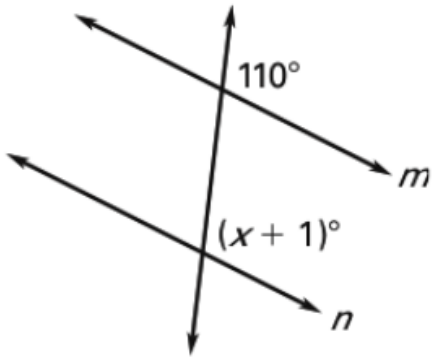


Find the value of x that makes $m \parallel n$.

4.



5.



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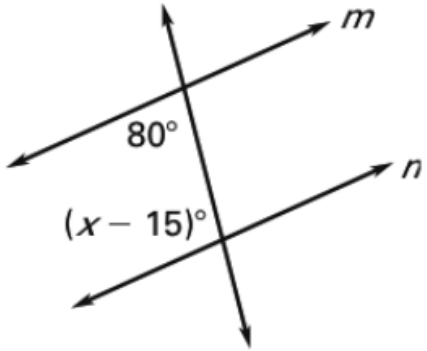

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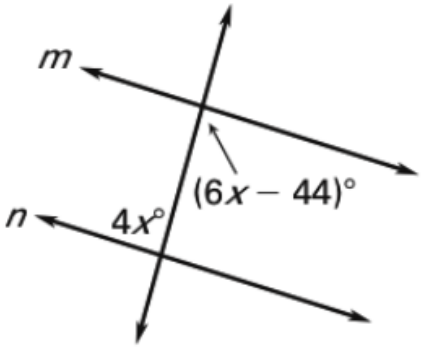
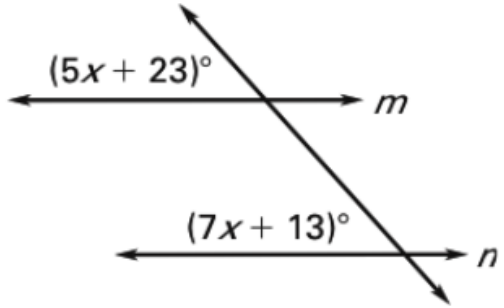
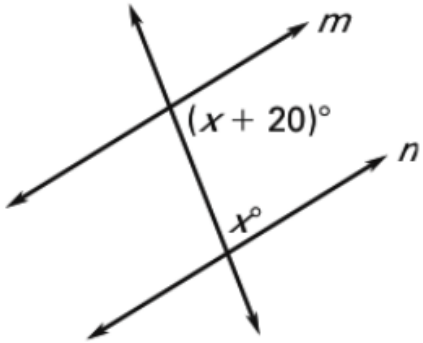


8.



9.



LESSON 3.3

Practice *continued*

For use with pages 161–169

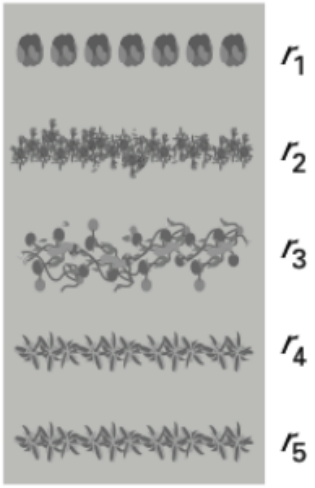
In Exercises 10–12, choose the word that best completes the statement.

10. If two lines are cut by a transversal so the alternate interior angles are (*congruent*, *supplementary*, *complementary*), then the lines are parallel.

11. If two lines are cut by a transversal so the consecutive interior angles are (*congruent*, *supplementary*, *complementary*), then the lines are parallel.

12. If two lines are cut by a transversal so the corresponding angles are (*congruent*, *supplementary*, *complementary*), then the lines are parallel.

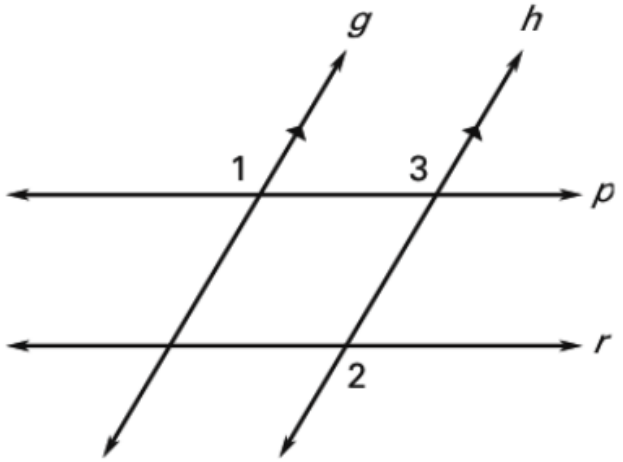
13. Gardens A garden has five rows of vegetables. Each row is parallel to the row immediately next to it. *Explain* why the first row is parallel to the last row.



In Exercises 14–18, complete the two-column proof.

GIVEN: $g \parallel h$, $\angle 1 \cong \angle 2$

PROVE: $p \parallel r$

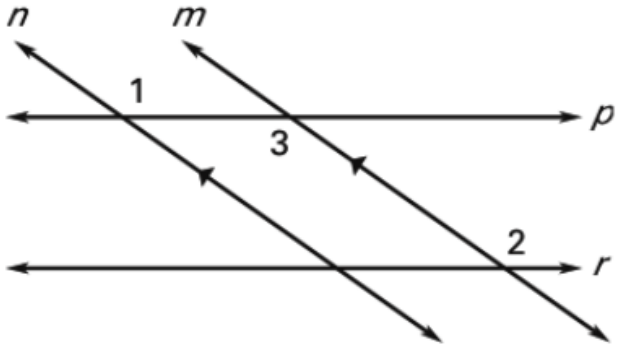


Statements	Reasons
$g \parallel h$	**14.** ?
$\angle 1 \cong \angle 3$	**15.** ?
$\angle 1 \cong \angle 2$	**16.** ?
$\angle 2 \cong \angle 3$	**17.** ?
$p \parallel r$	**18.** ?

In Exercises 19–23, complete the two-column proof.

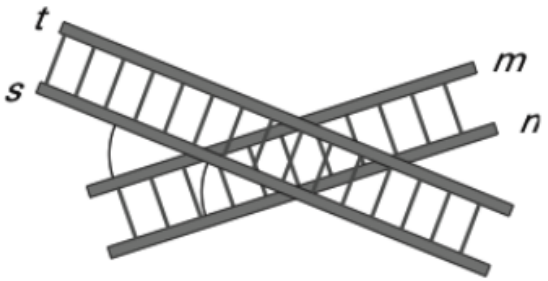
GIVEN: $n \parallel m$, $\angle 1 \cong \angle 2$

PROVE: $p \parallel r$



Statements	Reasons
$n \parallel m$	**19.** ?
$\angle 1 \cong \angle 3$	**20.** ?
$\angle 1 \cong \angle 2$	**21.** ?
$\angle 2 \cong \angle 3$	**22.** ?
$p \parallel r$	**23.** ?

24. Railroad Tracks Two sets of railroad tracks intersect as shown. How do you know that line n is parallel to line m?



Answer Key

Lesson 3.3

Practice Level B

1. yes; Consecutive Interior Angles Converse

2. yes; Alternate Interior Angles Converse

3. no **4.** 40 **5.** 109 **6.**115 **7.** 22 **8.** 5 **9.** 80

10. congruent **11.** supplementary **12.** congruent

13. Each row is parallel to the one next to it, so $r_1 \parallel r_2$, $r_2 \parallel r_3$, and so on. Then $r_1 \parallel r_3$ by the Transitive Property of Parallel Lines. By continuing this reasoning, $r_1 \parallel r_5$. So, the first row is parallel to the last row.

14. Given **15.** Corresponding Angles Postulate **16.** Given

17. Transitive Property of Equality **18.** Alternate Exterior Angles Converse **19.** Given

20. Alternate Interior Angles Theorem **21.** Given **22.** Transitive Property of Equality

23. Alternate Interior Angles Converse **24.** Corresponding Angles Converse

Day 2 Assignment:

p. 165 (3-15, 19-21, 31, 33, 34, 46-54)