4.1 Apply Triangle Sum Properties

Goal • Classsify triangles and find measures of their angles.

*triangle- a polygon with three sides

*A triangle with vertices A, B, and C is called:
"triangle ABC" or "$\triangle$ABC"

READ VOCABULARY

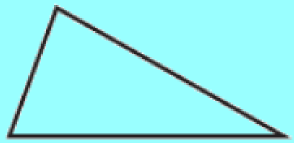
Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

KEY CONCEPT

For Your Notebook

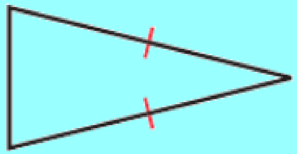
Classifying Triangles by Sides

Scalene Triangle



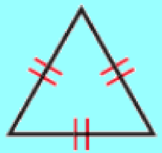
No congruent sides

Isosceles Triangle



At least 2 congruent sides

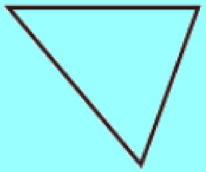
Equilateral Triangle



3 congruent sides

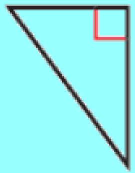
Classifying Triangles by Angles

Acute Triangle



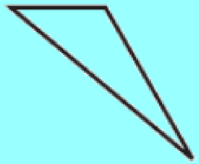
3 acute angles

Right Triangle



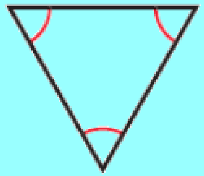
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle



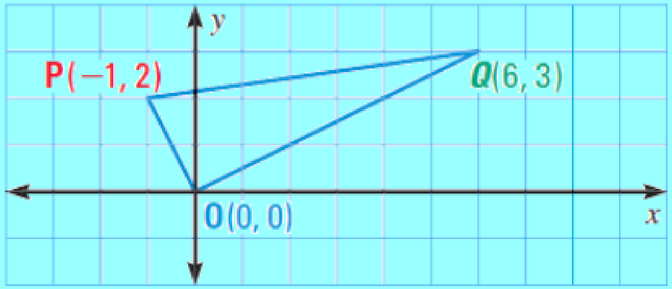
3 congruent angles

EXAMPLE 1 Classify triangles by sides and by angles

SUPPORT BEAMS Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.

EXAMPLE 2 Classify a triangle in a coordinate plane

Classify $\triangle PQO$ by its sides. Then determine if the triangle is a right triangle.



Solution

STEP 1 **Use** the distance formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-1) - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-1))^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

STEP 2 **Check** for right angles. The slope of $\overline{OP}$ is $\frac{2 - 0}{-1 - 0} = -2$. The slope of $\overline{OQ}$ is $\frac{3 - 0}{6 - 0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$, so $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

▶ Therefore, $\triangle PQO$ is a right scalene triangle.

Interior Angles vs. Exterior Angles

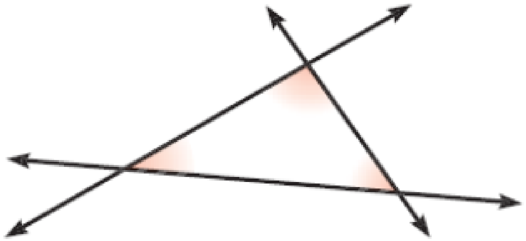
*The original angles "inside" the triangle are the **interior angles**.

*The angles that form linear pairs with the interior angles are the **exterior angles**.

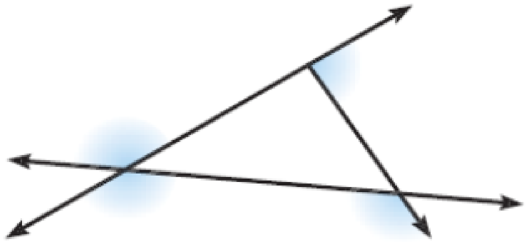
ANGLES When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

READ DIAGRAMS

Each vertex has a *pair* of congruent exterior angles. However, it is common to show only *one* exterior angle at each vertex.

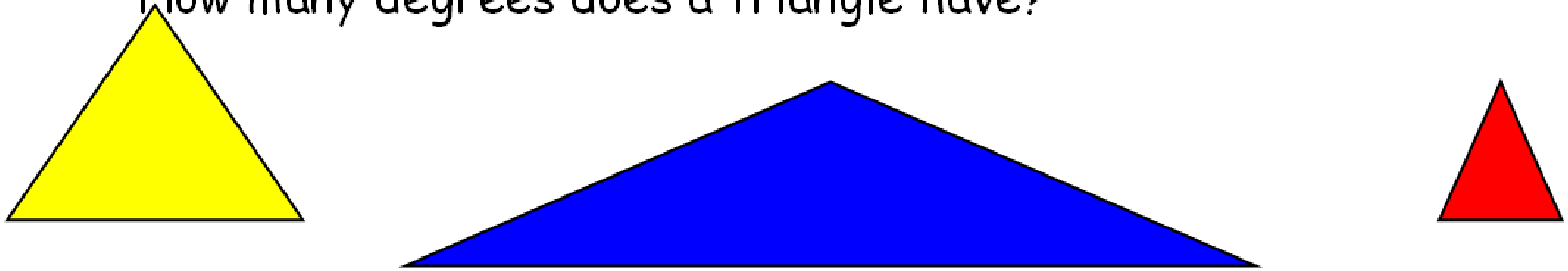


interior angles



exterior angles

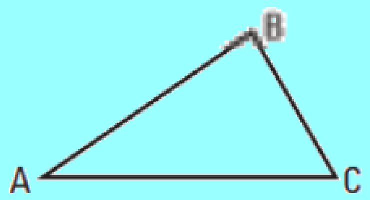
*How many degrees does a triangle have?



THEOREM *For Your Notebook*

THEOREM 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.



$m\angle A + m\angle B + m\angle C = 180°$

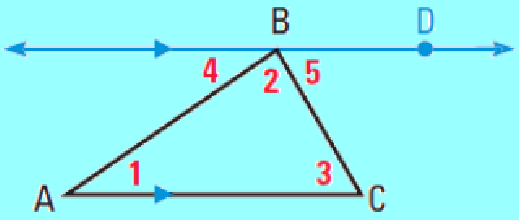
Proof: p. 219; Ex. 53, p. 224

AUXILIARY LINES To prove certain theorems, you may need to add a line, a segment, or a ray to a given diagram. An *auxiliary* line is used in the proof of the Triangle Sum Theorem.

PROOF Triangle Sum Theorem

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Plan for Proof

a. Draw an auxiliary line through B and parallel to $\overline{AC}$.

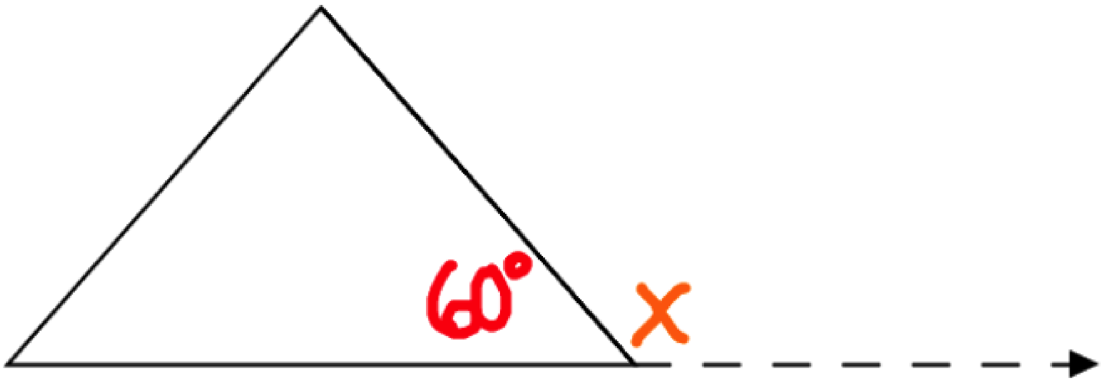
b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180°$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

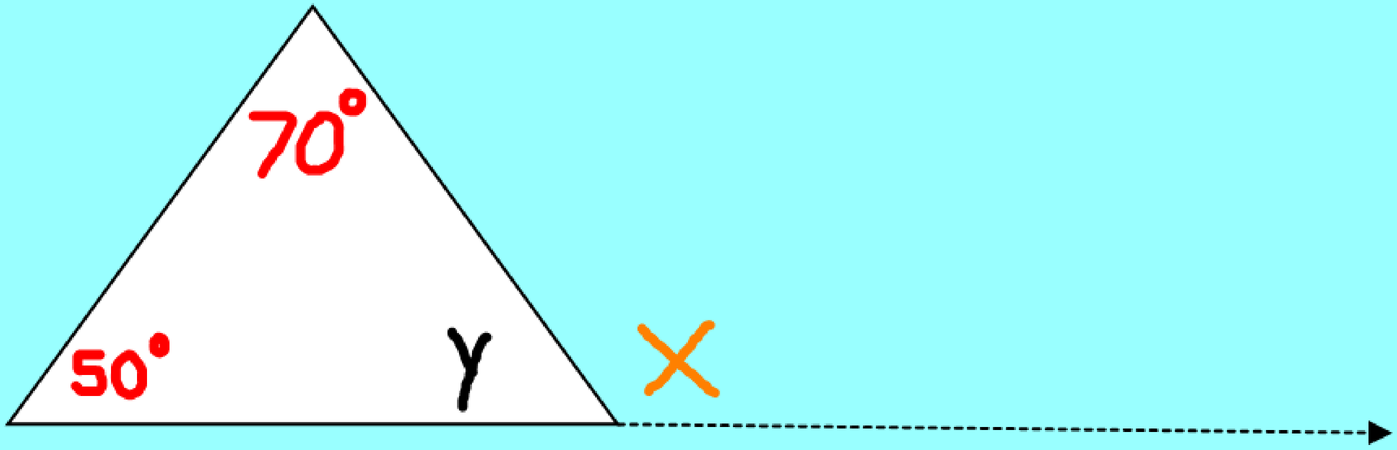
Plan in Action

	STATEMENTS	REASONS
a.	**1.** Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	**1.** Parallel Postulate
b.	**2.** $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	**2.** Angle Addition Postulate and definition of straight angle
	3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	**3.** Alternate Interior Angles Theorem
	4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	**4.** Definition of congruent angles
c.	**5.** $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	**5.** Substitution Property of Equality

How would we find x?



What if I give you this information?

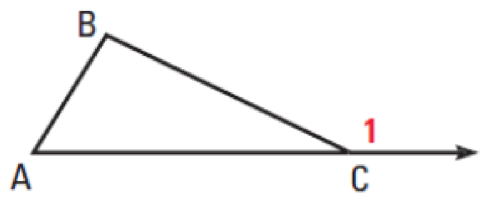


THEOREM *For Your Notebook*

THEOREM 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

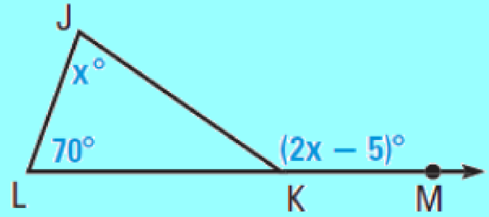
Proof: Ex. 50, p. 223



$m\angle 1 = m\angle A + m\angle B$

EXAMPLE 3 Find an angle measure

ALGEBRA Find $m\angle JKM$.



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$(2x - 5)° = 70° + x°$ **Apply the Exterior Angle Theorem.**

$x = 75$ **Solve for x.**

STEP 2 **Substitute** 75 for x in $2x - 5$ to find $m\angle JKM$.

$2x - 5 = 2 \cdot 75 - 5 = 145$

▶ The measure of $\angle JKM$ is 145°.

<u>Corollary to a theroem -</u> A statement that can be proven easily using the theorem_

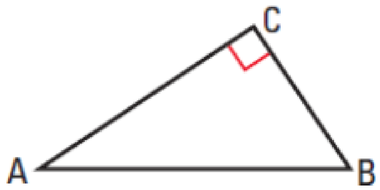
COROLLARY

For Your Notebook

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

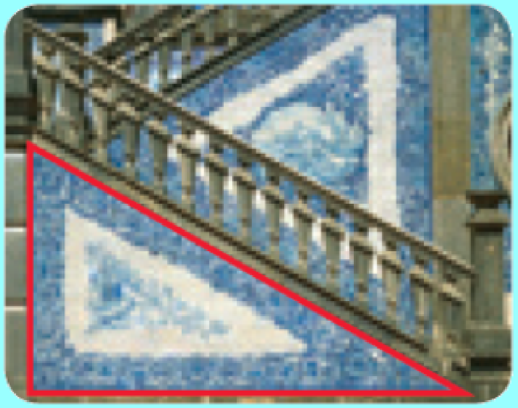
Proof: Ex. 48, p. 223



$m\angle A + m\angle B = 90°$

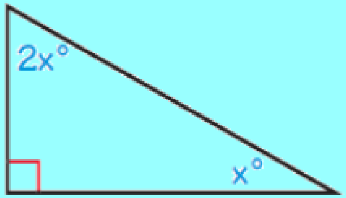
EXAMPLE 4 Find angle measures from a verbal description

ARCHITECTURE The tiled staircase shown forms a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.



Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.



Use the corollary to set up and solve an equation.

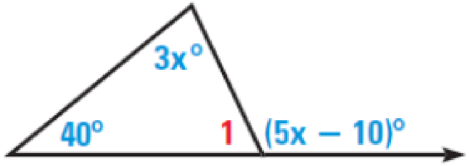
$x° + 2x° = 90°$ Corollary to the Triangle Sum Theorem

$x = 30$ Solve for *x*.

▶ So, the measures of the acute angles are 30° and 2(30°) = 60°.

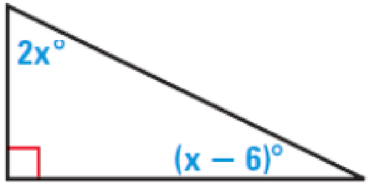
✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the measure of $\angle 1$ in the diagram shown.



4. Find the measure of each interior angle of $\triangle ABC$, where $m\angle A = x°$, $m\angle B = 2x°$, and $m\angle C = 3x°$.

5. Find the measures of the acute angles of the right triangle in the diagram shown.



6. In Example 4, what is the measure of the obtuse angle formed between the staircase and a segment extending from the horizontal leg?

Assignment:

p. 221 (1-19, 21-27, 32-37, 54-63 all)