4.4 Prove Triangles Congruent by SAS and HL

Goal • Use sides and angles to prove congruence.

Consider a relationship involving two sides and the angle they form, their *included* angle. To picture the relationship, form an angle using two pencils.

Any time you form an angle of the same measure with the pencils, the side formed by connecting the pencil points will have the same length. In fact, any two triangles formed in this way are congruent.

POSTULATE *For Your Notebook*

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

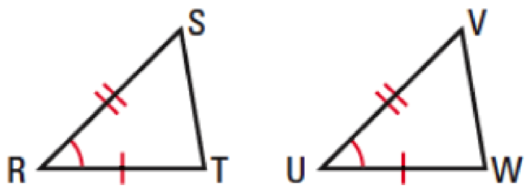
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{RS} \cong \overline{UV}$,

Angle $\angle R \cong \angle U$, and

Side $\overline{RT} \cong \overline{UW}$,

then $\triangle RST \cong \triangle UVW$.

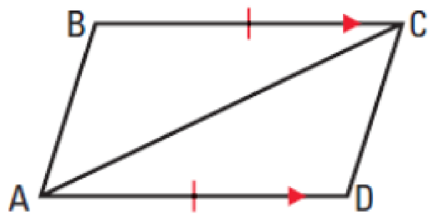


EXAMPLE 1 Use the SAS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\triangle ABC \cong \triangle CDA$

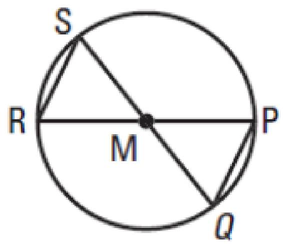


STATEMENTS	REASONS

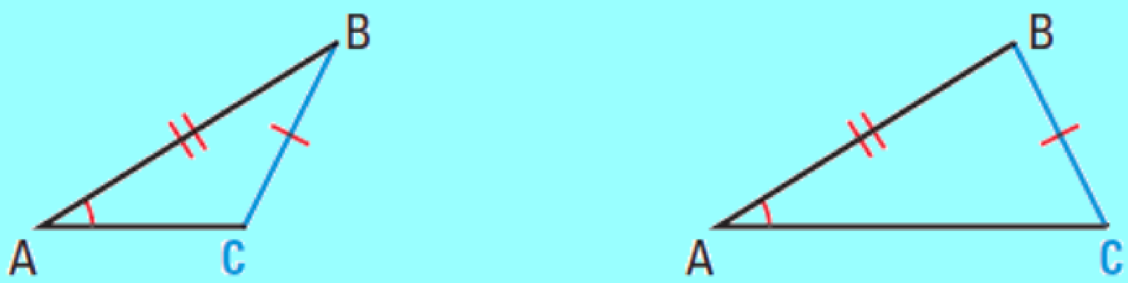
WRITE PROOFS
Make your proof easier to read by identifying the steps where you show congruent sides (S) and angles (A).

EXAMPLE 2 Use SAS and properties of shapes

In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?

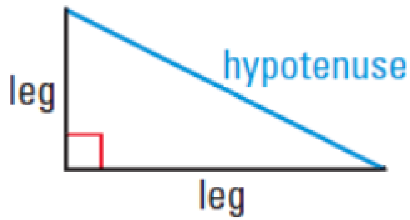


In general, if you know the lengths of two sides and the measure of an angle that is ***not included*** between them, you can create two different triangles.



Therefore, SSA is NOT a valid method for proving that triangles are congruent, although there is a special case for right triangles.

RIGHT TRIANGLES In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.

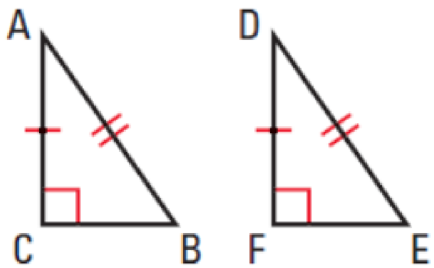


THEOREM

For Your Notebook

THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.



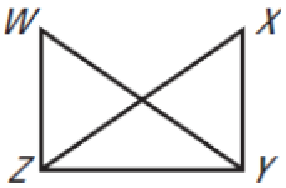
$\triangle ABC \cong \triangle DEF$

Proofs: Ex. 37, p. 439; p. 932

EXAMPLE 3 Use the Hypotenuse-Leg Congruence Theorem

USE DIAGRAMS
If you have trouble matching vertices to letters when you separate the overlapping triangles, leave the triangles in their original orientations.

Write a proof.

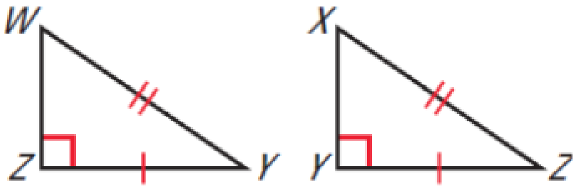


GIVEN ▶ $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

PROVE ▶ $\triangle WYZ \cong \triangle XZY$

Solution

Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



STATEMENTS	REASONS

Animated Geometry at classzone.com

EXAMPLE 4 Choose a postulate or theorem

SIGN MAKING You are making a canvas sign to hang on the triangular wall over the door to the barn shown in the picture. You think you can use two identical triangular sheets of canvas. You know that $\overline{RP} \perp \overline{QS}$ and $\overline{PQ} \cong \overline{PS}$. What postulate or theorem can you use to conclude that $\triangle PQR \cong \triangle PSR$?

Day 1 Assignment:

4.4 ws

LESSON 4.4

Practice B

For use with pages 240–247

Use the diagram to name the included angle between the given pair of sides.

1. $\overline{AB}$ and $\overline{BC}$

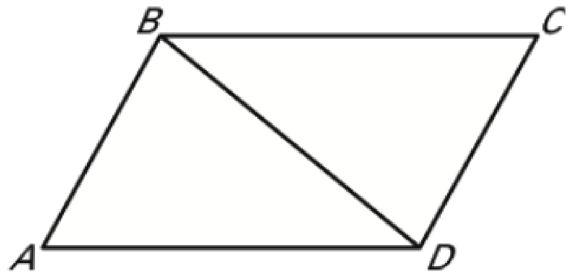
2. $\overline{BC}$ and $\overline{CD}$

3. $\overline{AB}$ and $\overline{BD}$

4. $\overline{BD}$ and $\overline{DA}$

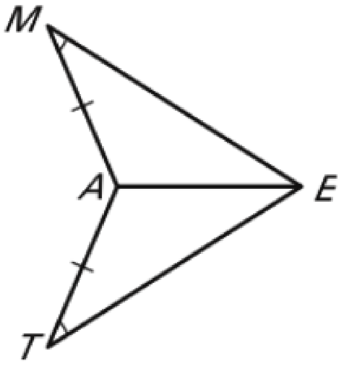
5. $\overline{DA}$ and $\overline{AB}$

6. $\overline{CD}$ and $\overline{DB}$

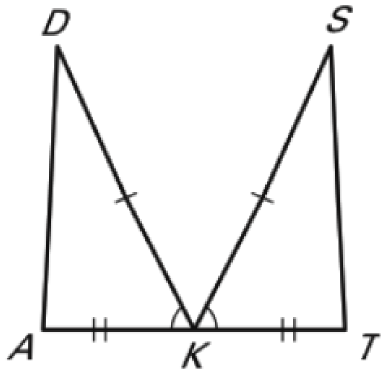


Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Postulate.

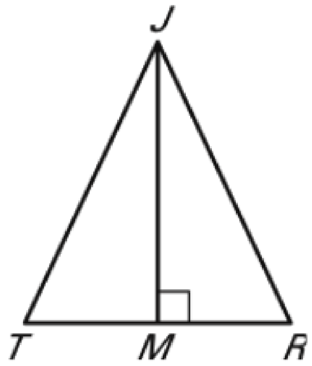
7. $\triangle MAE, \triangle TAE$



8. $\triangle DKA, \triangle TKS$

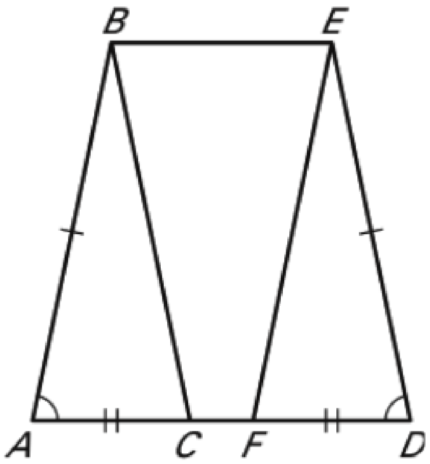


9. $\triangle JRM, \triangle JTM$

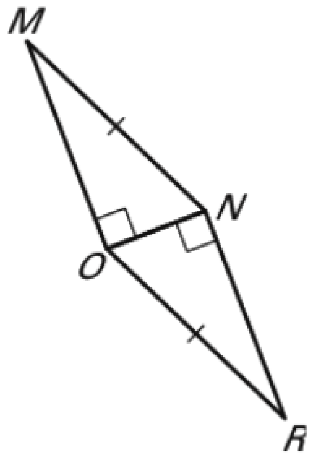


Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.

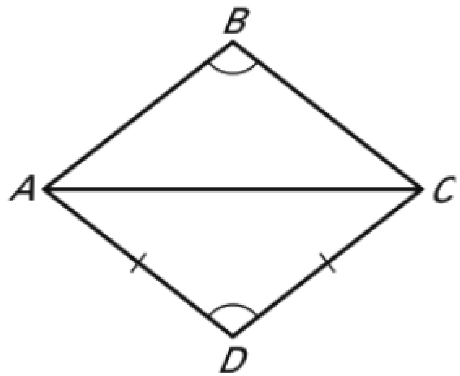
10. $\triangle ABC, \triangle DEF$



11. $\triangle MNO, \triangle RON$



12. $\triangle ABC, \triangle ADC$

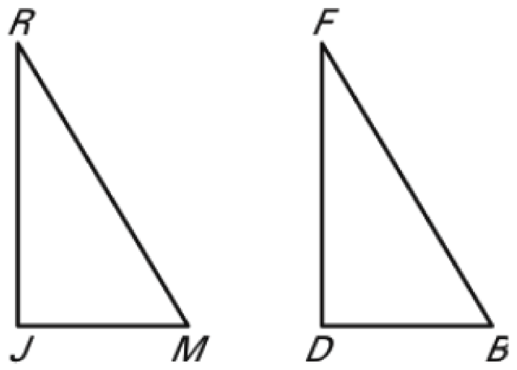


State the third congruence that must be given to prove that $\triangle JRM \cong \triangle DFB$ using the indicated postulate.

13. GIVEN: $\overline{JR} \cong \overline{DF}$, $\overline{JM} \cong \overline{DB}$, ___?___ $\cong$ ___?___
Use the SSS Congruence Postulate.

14. GIVEN: $\overline{JR} \cong \overline{DF}$, $\overline{JM} \cong \overline{DB}$, ___?___ $\cong$ ___?___
Use the SAS Congruence Postulate.

15. GIVEN: $\overline{RM} \cong \overline{FB}$, $\angle J$ is a right angle and $\angle J \cong \angle D$, ___?___ $\cong$ ___?___
Use the HL Congruence Theorem.



LESSON 4.4

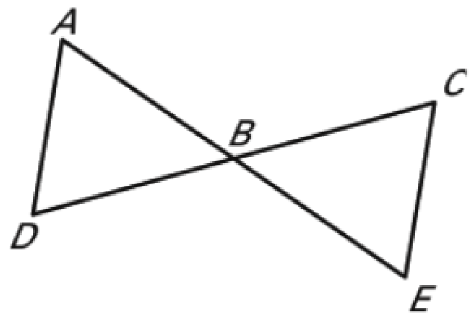
Practice B *continued*

For use with pages 240–247

16. Proof Complete the proof.

GIVEN: B is the midpoint of $\overline{AE}$.
B is the midpoint of $\overline{CD}$.

PROVE: $\triangle ABD \cong \triangle EBC$

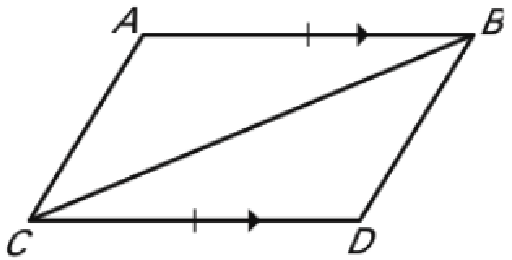


Statements	Reasons
1. B is the midpoint of $\overline{AE}$.	**1.** ?
2. ?	**2.** Definition of midpoint
3. B is the midpoint of $\overline{CD}$.	**3.** ?
4. ?	**4.** Definition of midpoint
5. $\angle ABD \cong \angle EBC$	**5.** ?
6. $\triangle ABD \cong \triangle EBC$	**6.** ?

17. Proof Complete the proof.

GIVEN: $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$

PROVE: $\triangle ABC \cong \triangle DCB$



Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	1. ?
2. $\angle ABC \cong \angle DCB$	2. ?
3. $\overline{AB} \cong \overline{CD}$	3. ?
4. $\overline{CB} \cong \overline{CB}$	4. ?
5. $\triangle ABC \cong \triangle DCB$	5. ?

Answer Key

Lesson 4.4

Practice Level B

1. $\angle ABC$ **2.** $\angle BCD$ **3.** $\angle ABD$ **4.** $\angle BDA$

5. $\angle DAB$ **6.** $\angle CDB$ **7.** not enough **8.** enough

9. not enough **10.** Yes, SAS Congruence Postulate **11.** Yes, HL Congruence Theorem

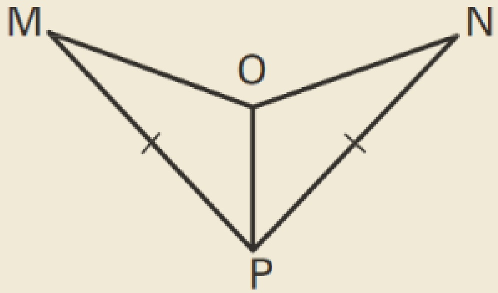
12. not enough **13.** $\overline{RM} \cong \overline{FB}$ **14.** $\angle J \cong \angle D$

15. $\overline{JM} \cong \overline{DB}$ or $\overline{JR} \cong \overline{DF}$ **16.** Given; $\overline{AB} \cong \overline{BE}$; Given; $\overline{CB} \cong \overline{BD}$; Vertical Angles Theorem; SAS Congruence Postulate **17.** Given; Alternate Interior Angles Theorem; Given; Reflexive Property of Congruence; SAS Congruence Postulate

Extra Example 1

Given: $\overline{MP} \cong \overline{NP}$, $\overline{OP}$ bisects $\angle MPN$.

Prove: $\triangle MOP \cong \triangle NOP$



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Day 2 Assignment:

p. 244 (1-15 odd, 20-22, 25-28, 31, 32, 34-36, 42-48)