

5.1**Midsegment Theorem and Coordinate Proof****Goal**

- Use properties of midsegments and write coordinate proofs.

Midsegment of a triangle-Is a segment that connects the midpoints of two sides of the triangle

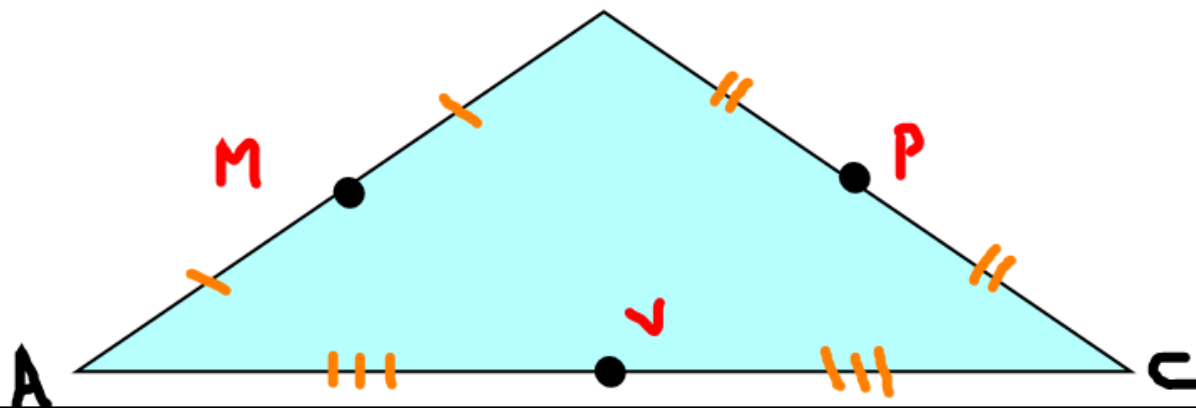
("Midsegment" think of Middle segment!)

*Every triangle has _____ midsegments??

(0, 1, 2, 3, 4, 5, 6, 7, ...???)

(0, 1, 2, 3, 4, 5, 6, 7, ...???)

Remember:
Tick Marks!!

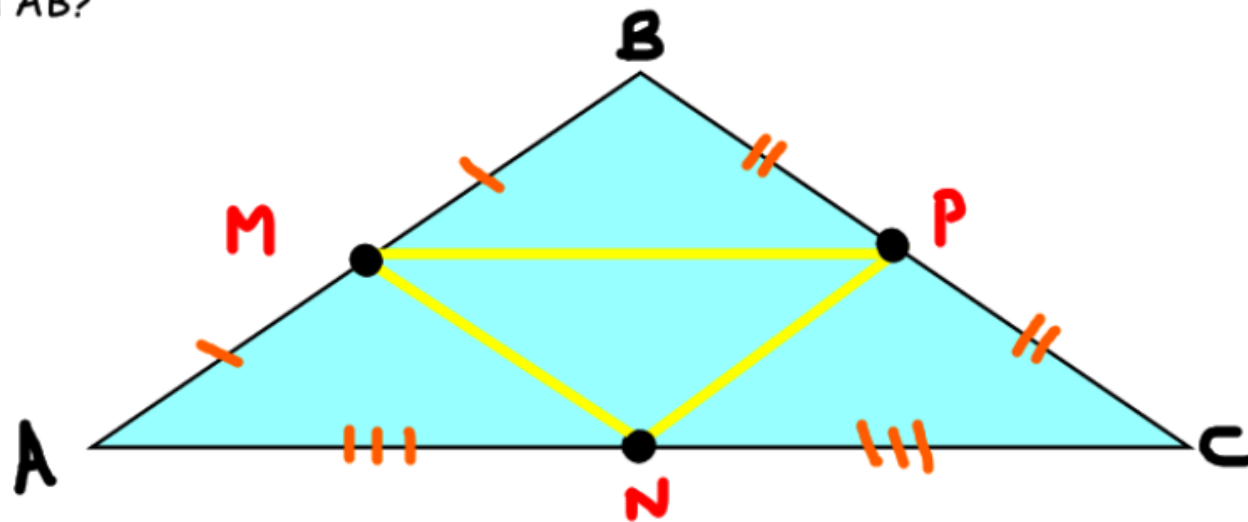


Let's investigate this a little more:

Is there a relationship between MP and AC ?

If so, what relationship(s) can you find?

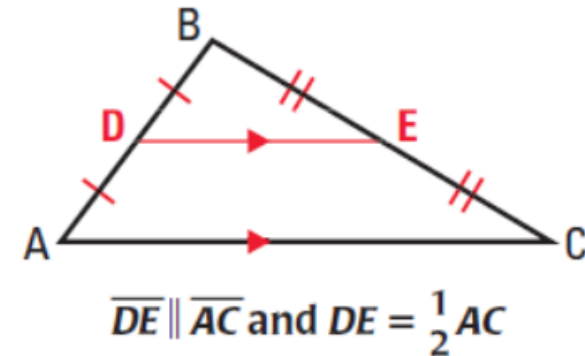
Is there a relationship between MN and BC ?
between NP and AB ?



THEOREM*For Your Notebook***THEOREM 5.1** Midsegment Theorem

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

Proof: Example 5, p. 297; Ex. 41, p. 300

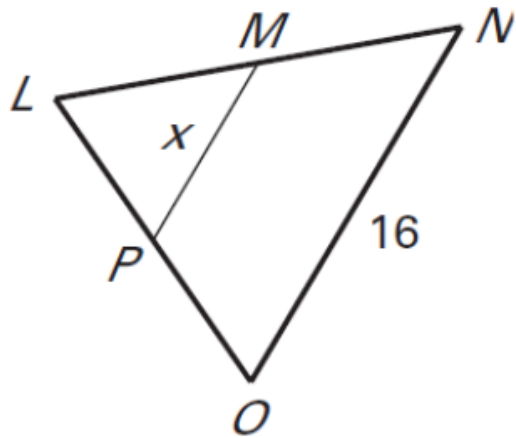
**AGAIN!!!!**

Midsegments are **PARALLEL** to a side.

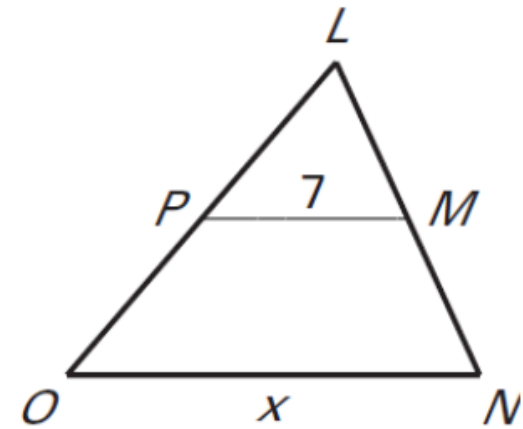
Midsegments are **HALF** as long as the side it is parallel to.

\overline{MP} is a midsegment of $\triangle LNO$. Find the value of x .

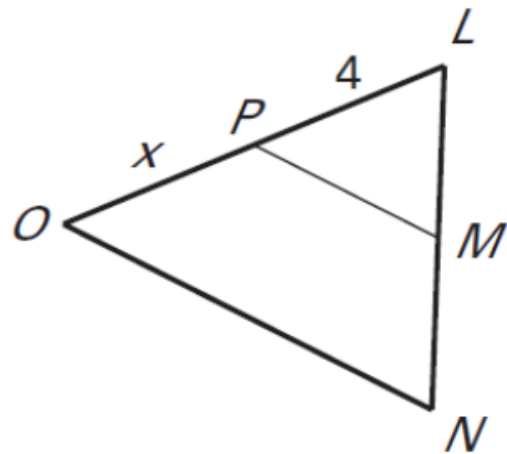
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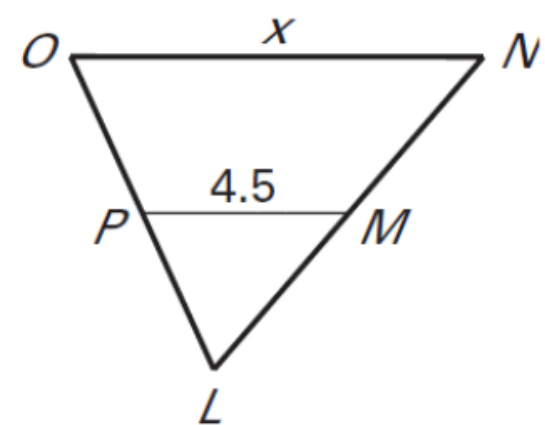
2.



3.



4.



EXAMPLE 1**Use the Midsegment Theorem to find lengths****READ DIAGRAMS**

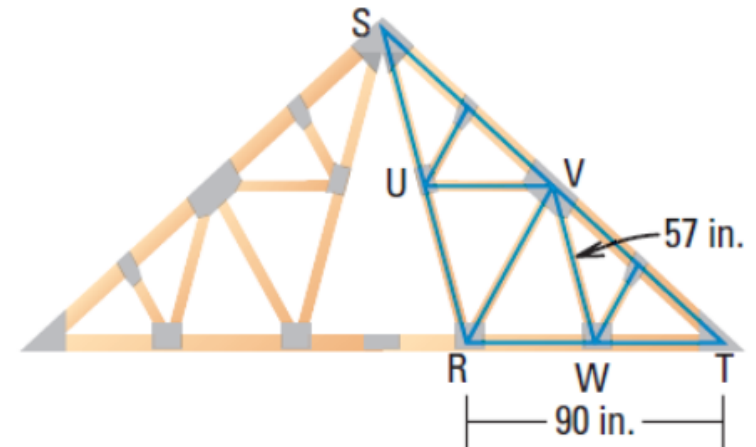
In the diagram for Example 1, midsegment \overline{UV} can be called "the midsegment opposite \overline{RT} ."

CONSTRUCTION Triangles are used for strength in roof trusses. In the diagram, \overline{UV} and \overline{VW} are midsegments of $\triangle RST$. Find UV and RS .

Solution

$$UV = \frac{1}{2} \cdot RT = \frac{1}{2}(90 \text{ in.}) = 45 \text{ in.}$$

$$RS = 2 \cdot VW = 2(57 \text{ in.}) = 114 \text{ in.}$$

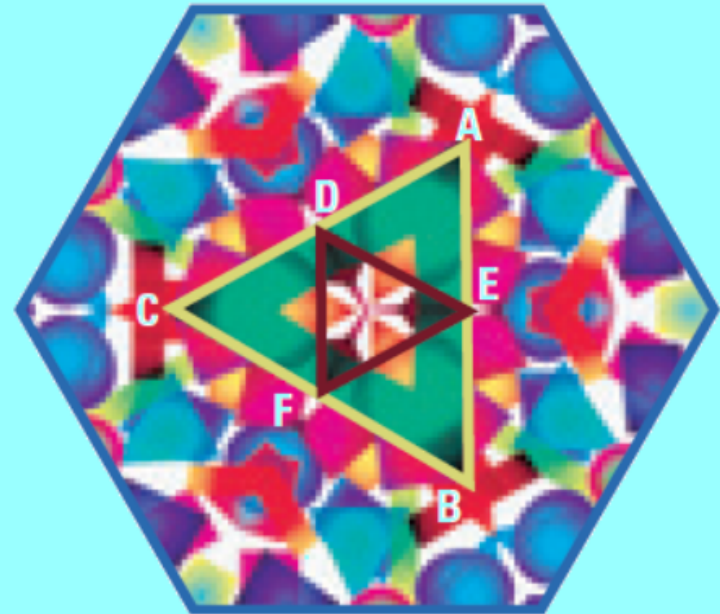


EXAMPLE 2 Use the Midsegment Theorem

In the kaleidoscope image, $\overline{AE} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CD}$. Show that $\overline{CB} \parallel \overline{DE}$.

Solution

Because $\overline{AE} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CD}$, E is the midpoint of \overline{AB} and D is the midpoint of \overline{AC} by definition. Then \overline{DE} is a midsegment of $\triangle ABC$ by definition and $\overline{CB} \parallel \overline{DE}$ by the Midsegment Theorem.



Coordinate Proof

a type of proof, which involves placing geometric figures in a coordinate plane

When you use variables to represent the coordinates of a figure in a coordinate proof, the results are true for all figures of that type.

EXAMPLE 3 Place a figure in a coordinate plane

Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.



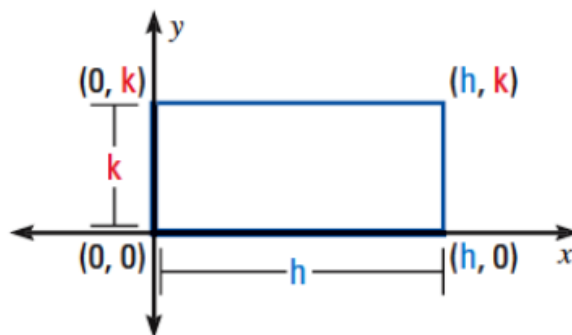
a. A rectangle

b. A scalene triangle

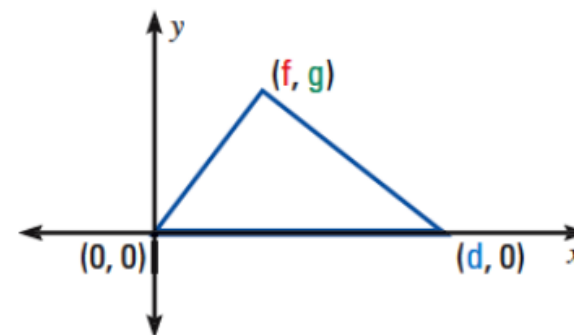
Solution

It is easy to find lengths of horizontal and vertical segments and distances from $(0, 0)$, so place one vertex at the origin and one or more sides on an axis.

a. Let h represent the length and k represent the width.



b. Notice that you need to use three different variables.



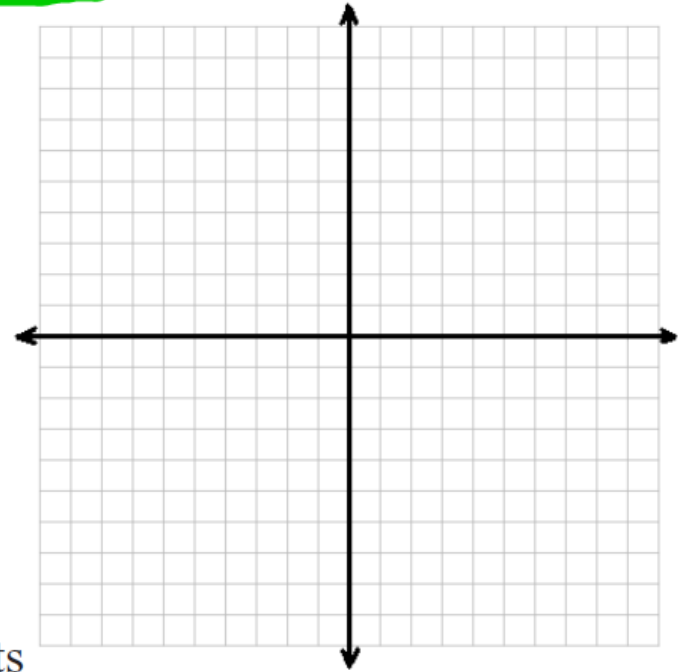
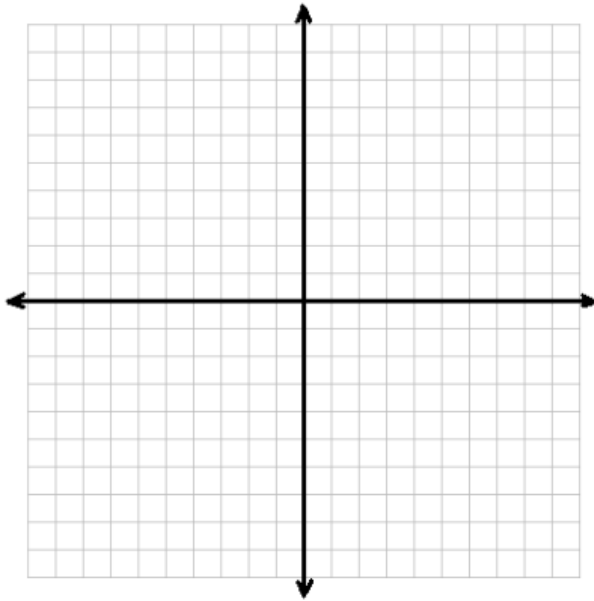
USE VARIABLES

The rectangle shown represents a general rectangle because the choice of coordinates is based only on the definition of a rectangle. If you use this rectangle to prove a result, the result will be true for all rectangles.

Place the figure in a coordinate plane. Assign coordinates to each vertex.
Explain the advantage of your placement.

15. Right triangle: leg lengths are 5 units and 9 units

1.



2 Isosceles right triangle: leg length is 14 units

Summary:

1. A triangle has _____ midsegments.
2. Each midsegment is _____ to one of the sides.
3. Each midsegment is _____ the length of one of the sides.
4. Coordinate proof involves placing geometric figures in a _____.

Day 1 Assignment:

5.1 ws

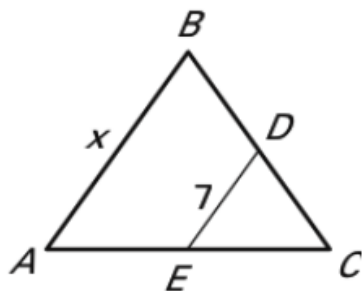
LESSON
5.1

Practice B

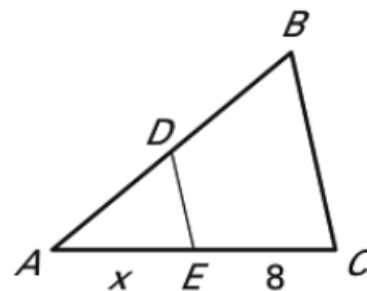
For use with pages 294–301

\overline{DE} is a midsegment of $\triangle ABC$. Find the value of x .

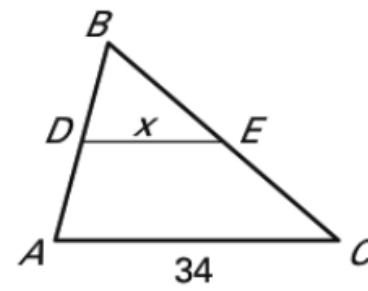
1.



2.



3.



In $\triangle JKL$, $\overline{JR} \cong \overline{RK}$, $\overline{KS} \cong \overline{SL}$, and $\overline{JT} \cong \overline{TL}$. Copy and complete the statement.

4. $\overline{RS} \parallel$?

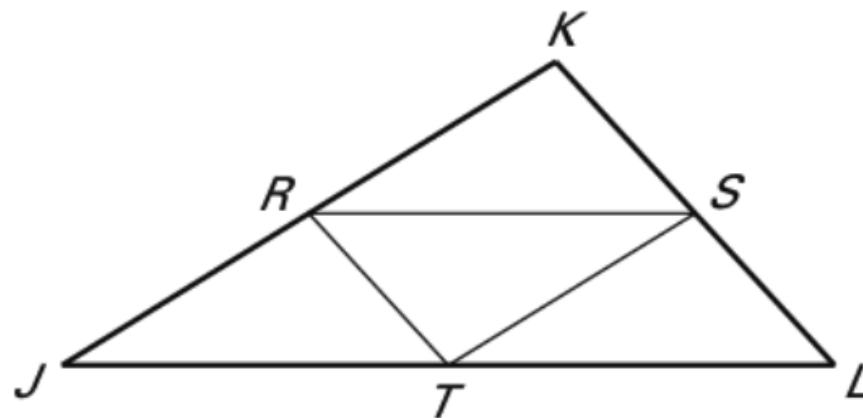
5. $\overline{ST} \parallel$?

6. $\overline{KL} \parallel$?

7. $\overline{SL} \cong$? \cong ?

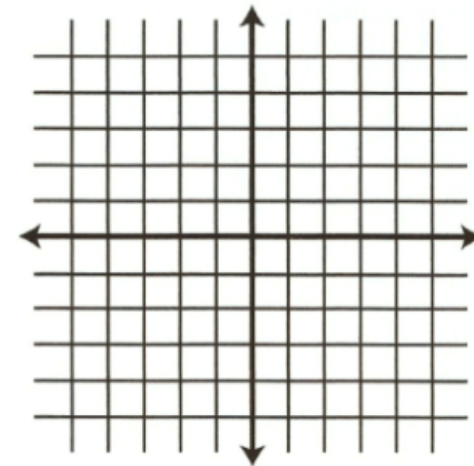
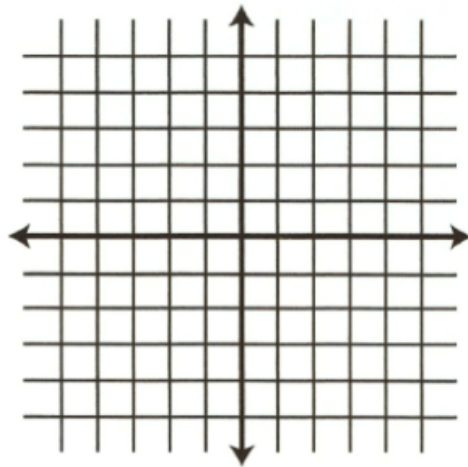
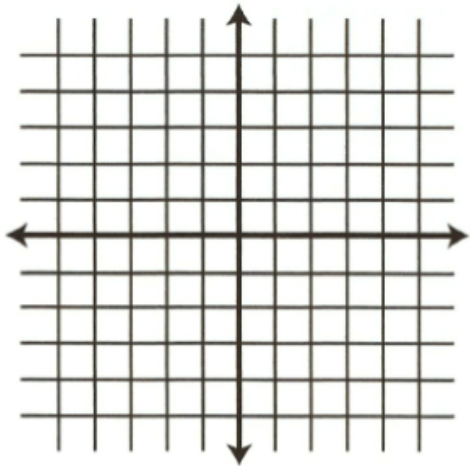
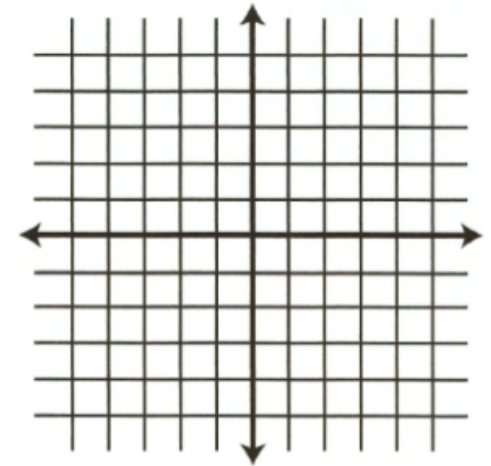
8. $\overline{JR} \cong$? \cong ?

9. $\overline{JT} \cong$? \cong ?



Place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex.

- 10.** Right triangle: leg lengths are 5 units and 3 units
- 11.** Rectangle: length is 7 units and width is 4 units
- 12.** Square: side length is 6 units
- 13.** Isosceles right triangle: leg length is 12 units

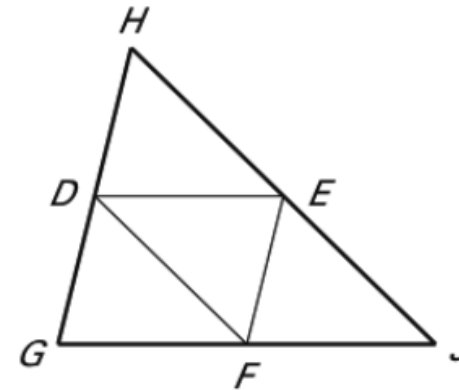


Use $\triangle GHJ$, where D , E , and F are midpoints of the sides.

14. If $DE = 4x + 5$ and $GJ = 3x + 25$, what is DE ?

15. If $EF = 2x + 7$ and $GH = 5x - 1$, what is EF ?

16. If $HJ = 8x - 2$ and $DF = 2x + 11$, what is HJ ?

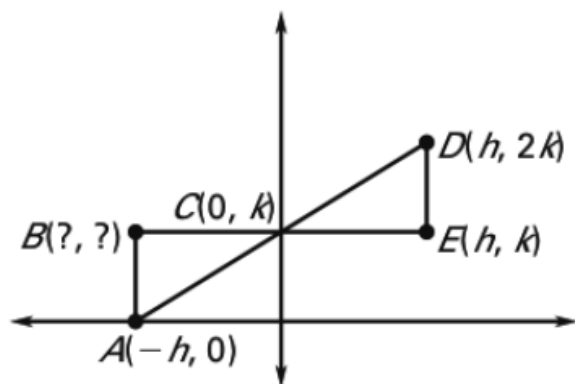


LESSON
5.1

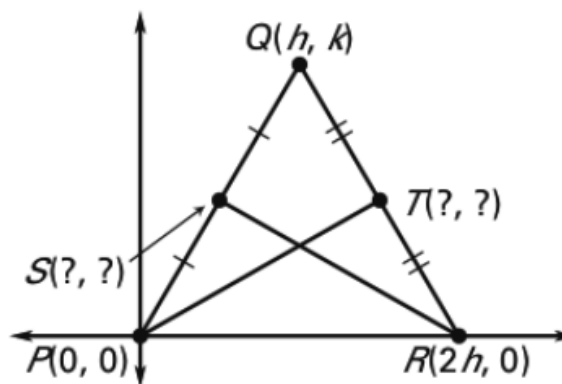
Practice B *continued*
For use with pages 294–301

Find the unknown coordinates of the point(s) in the figure. Then show that the given statement is true.

17. $\triangle ABC \cong \triangle DEC$

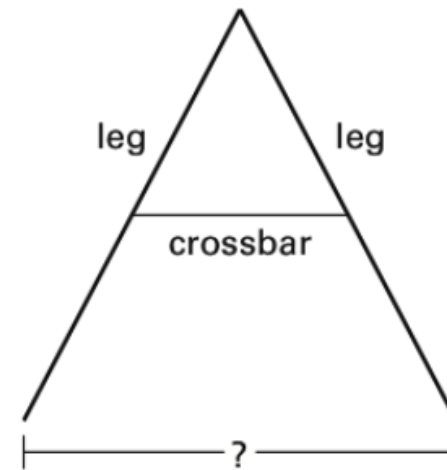


18. $\overline{PT} \cong \overline{SR}$

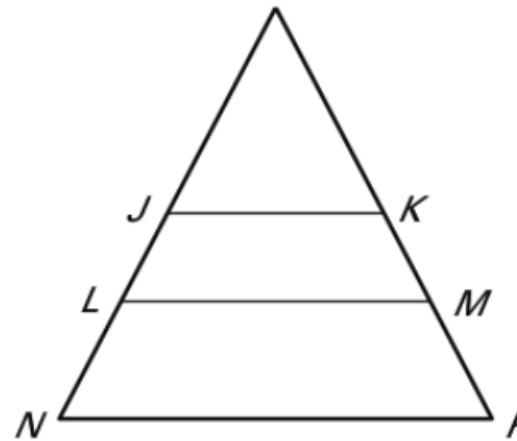


- 19.** The coordinates of $\triangle ABC$ are $A(0, 5)$, $B(8, 20)$, and $C(0, 26)$. Find the length of each side and the perimeter of $\triangle ABC$. Then find the perimeter of the triangle formed by connecting the three midsegments of $\triangle ABC$.

- 20. Swing Set** You are assembling the frame for a swing set. The horizontal crossbars in the kit you purchased are each 36 inches long. You attach the crossbars at the midpoints of the legs. At each end of the frame, how far apart will the bottoms of the legs be when the frame is assembled? *Explain.*



- 21. A-Frame House** In an A-frame house, the floor of the second level, labeled \overline{LM} , is closer to the first floor, \overline{NP} , than midsegment \overline{JK} . If \overline{JK} is 14 feet long, can \overline{LM} be 12 feet long? 14 feet long? 20 feet long? 24 feet long? 30 feet long? *Explain.*



Answer Key

Lesson 5.1

Practice Level B

1. 14 2. 8 3. 17 4. \overline{JL} 5. \overline{JK} 6. \overline{RT}

7. \overline{KS} , \overline{RT} 8. \overline{KR} , \overline{ST} 9. \overline{LT} , \overline{RS}

10. *Sample answer:* (0, 0), (5, 0), (0, 3)

11. *Sample answer:* (0, 0), (7, 0), (7, 4), (0, 4)

12. (0, 0), (6, 0), (6, 6), (0, 6)

13. (0, 0), (12, 0), (0, 12)

14. 17 15. 37 16. 46

17. $B(-h, k)$; Proof 18. $S\left(\frac{h}{2}, \frac{k}{2}\right)$, $T\left(\frac{3h}{2}, \frac{k}{2}\right)$; Proof

19. $AB = 17$, $BC = 10$, $AC = 21$; 48; 24

20. 72 in.; The crossbar is the midsegment of the legs. 21. no; no; yes; yes; no; $14 < LM < 28$

Day 2 Assignment:

p. 298 (3-19, 24-27, 36, 37, 47-52 all)