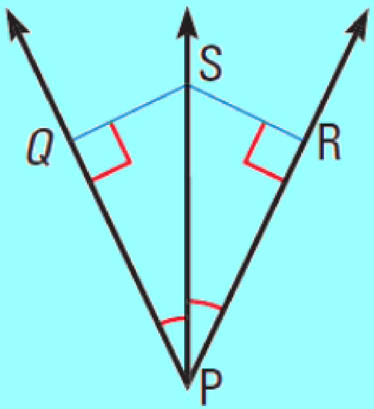
5.3 Use Angle Bisectors of Triangles

Goal • Use angle bisectors to find distance relationships.

Angle Bisector

What is it? What does it do?

angle bisector- A ray that divides an angle into two congruent adjacent angles.



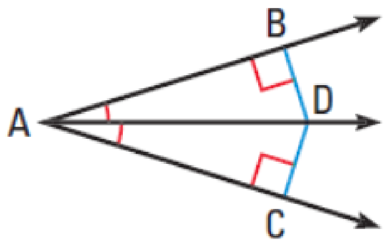
Remember: the distance from a point to a line is the length of the perpendicular segment from the point to the line.

THEOREMS — *For Your Notebook*

THEOREM 5.5 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.

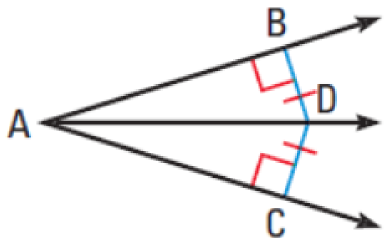


Proof: Ex. 34, p. 315

THEOREM 5.6 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

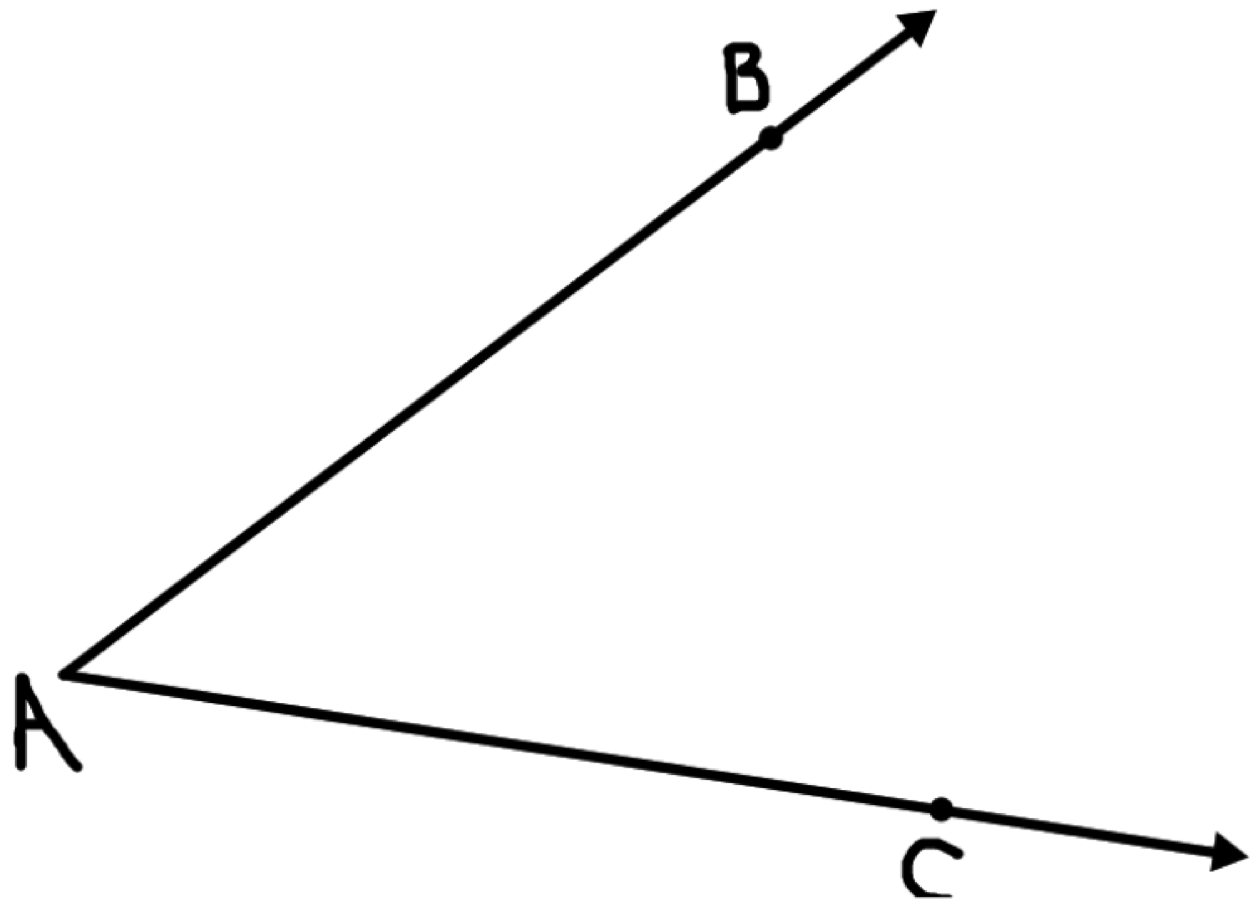


Proof: Ex. 35, p. 315

REVIEW DISTANCE

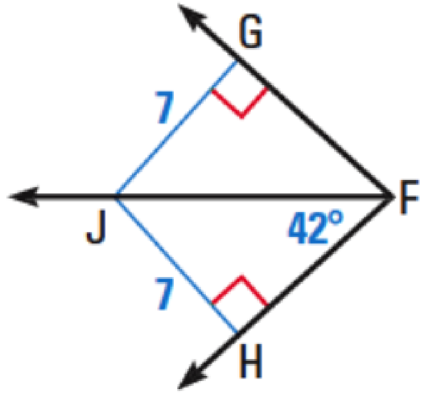
In Geometry, *distance* means the *shortest* length between two objects.

DRAW and LABEL what we know about an angle bisector.



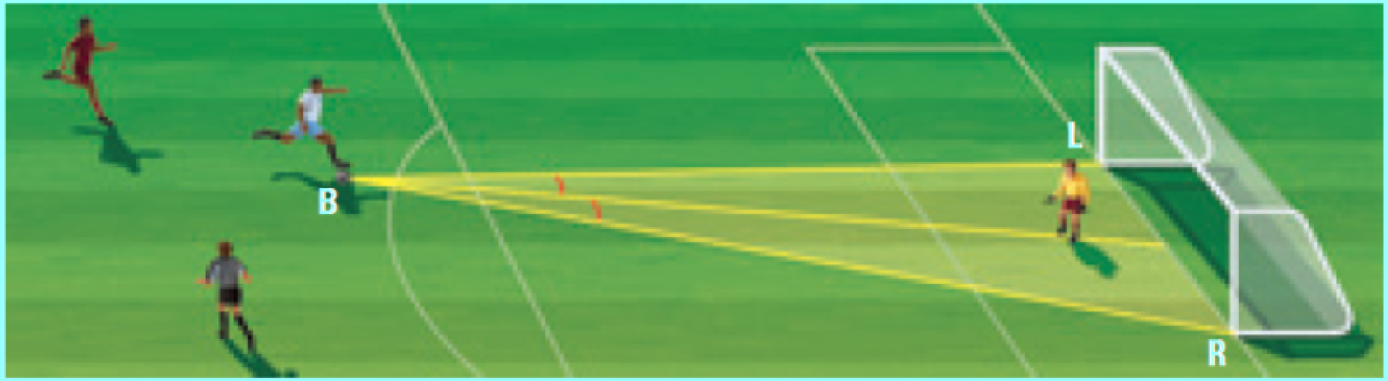
EXAMPLE 1 Use the Angle Bisector Theorems

Find the measure of $\angle GFJ$.



EXAMPLE 2 Solve a real-world problem

SOCCER A soccer goalie's position relative to the ball and goalposts forms congruent angles, as shown. Will the goalie have to move farther to block a shot toward the right goalpost R or the left goalpost L?



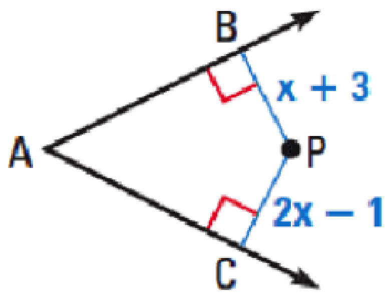
Solution

The congruent angles tell you that the goalie is on the bisector of $\angle LBR$. By the Angle Bisector Theorem, the goalie is equidistant from $\overrightarrow{BR}$ and $\overrightarrow{BL}$.

▸ So, the goalie must move the same distance to block either shot.

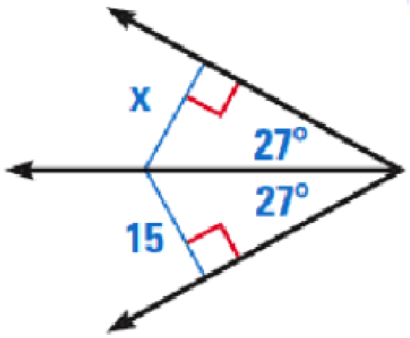
EXAMPLE 3 Use algebra to solve a problem

ALGEBRA For what value of x does P lie on the bisector of $\angle A$?



In Exercises 1–3, find the value of x.

1.



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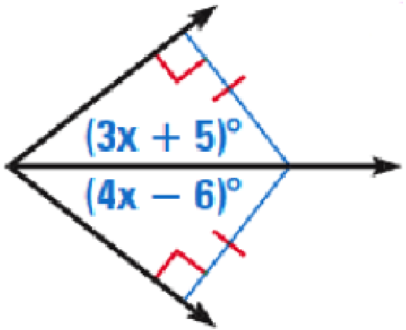
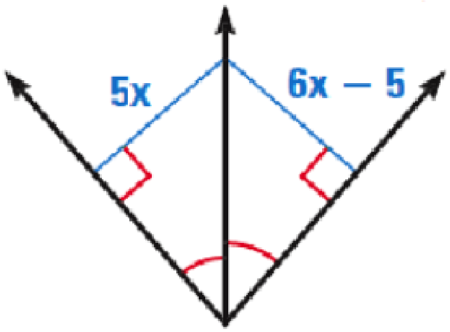
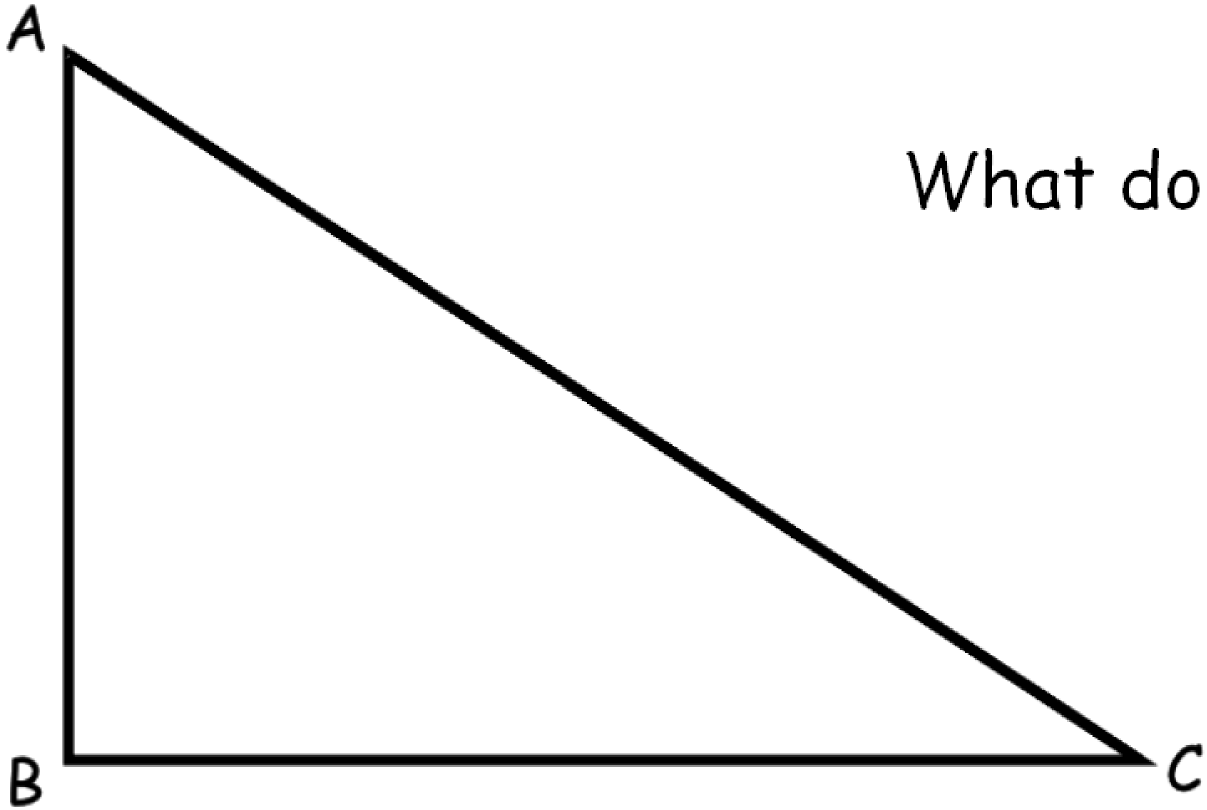


3.



Let's draw in all of the angle bisectors.

What do we find out?



READ VOCABULARY

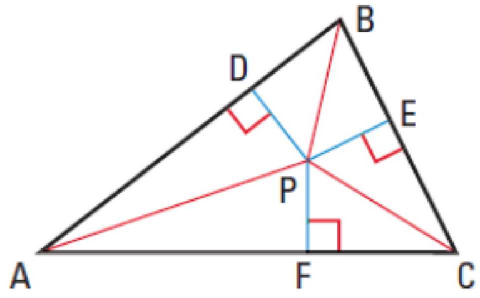
An *angle bisector of a triangle* is the bisector of an interior angle of the triangle.

THEOREM *For Your Notebook*

THEOREM 5.7 Concurrency of Angle Bisectors of a Triangle

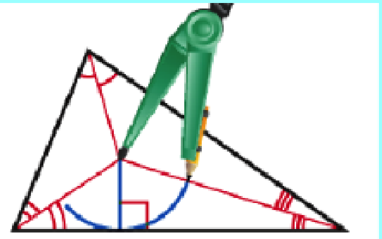
The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.



Proof: Ex. 36, p. 316

incenter- The point of concurrency of the three angle bisectors of a triangle

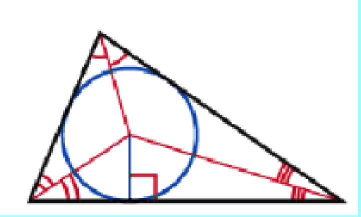


*Always lies **Inside** the triangle

*__Equidistant__ from the three sides of the triangle.

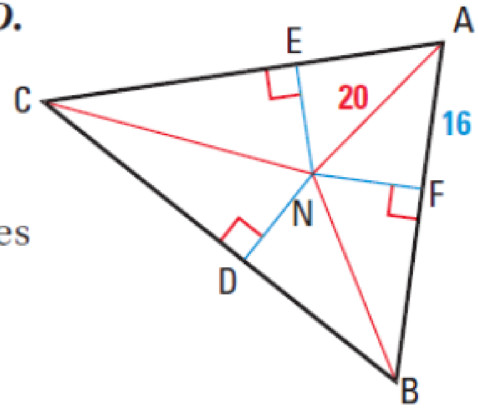
**We can draw a circle using the incenter as the center of the circle and the distance from one side as the radius_

Inscribed Circle- a circle within a figure



EXAMPLE 4 Use the concurrency of angle bisectors

In the diagram, N is the incenter of $\triangle ABC$. Find ND.



Solution

By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter N is equidistant from the sides of $\triangle ABC$. So, to find ND, you can find NF in $\triangle NAF$. Use the Pythagorean Theorem stated on page 18.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$20^2 = NF^2 + 16^2$	Substitute known values.
$400 = NF^2 + 256$	Multiply.
$144 = NF^2$	Subtract 256 from each side.
$12 = NF$	Take the positive square root of each side.

▶ Because $NF = ND$, $ND = 12$.

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REVIEW QUADRATIC EQUATIONS

For help with solving a quadratic equation by taking square roots, see page 882. Use only the positive square root when finding a distance, as in Example 4.

Assignment:

p. 313 (3-25, 42-44 all)