

5.4 Use Medians and Altitudes

Goal • Use medians and altitudes of triangles.

Try balancing a paper triangle on a pencil.



Three medians meet at the centroid.

Median of a triangle-A segment from the vertex to the midpoint of the opposite side

- Centroid-The point where the three medians of a triangle are concurrent. The centroid is inside of the triangle.

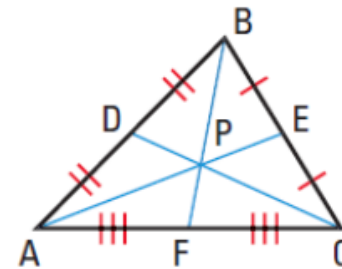
THEOREM*For Your Notebook***THEOREM 5.8** Concurrency of Medians of a Triangle

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

The medians of $\triangle ABC$ meet at P and

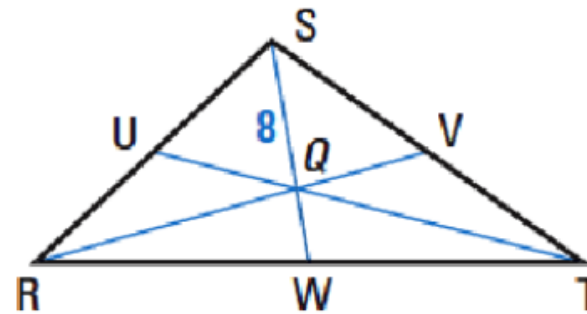
$$AP = \frac{2}{3}AE, BP = \frac{2}{3}BF, \text{ and } CP = \frac{2}{3}CD.$$

Proof: Ex. 32, p. 323; p. 934



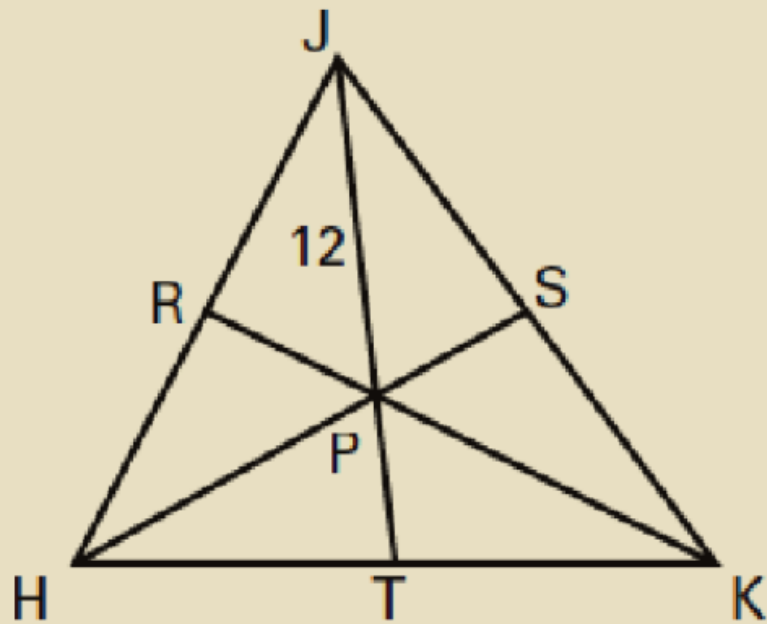
EXAMPLE 1 Use the centroid of a triangle

In $\triangle RST$, Q is the centroid and $SQ = 8$.
Find QW and SW .



Extra Example 1

In $\triangle HJK$, P is the centroid and $JP = 12$. Find PT and JT .

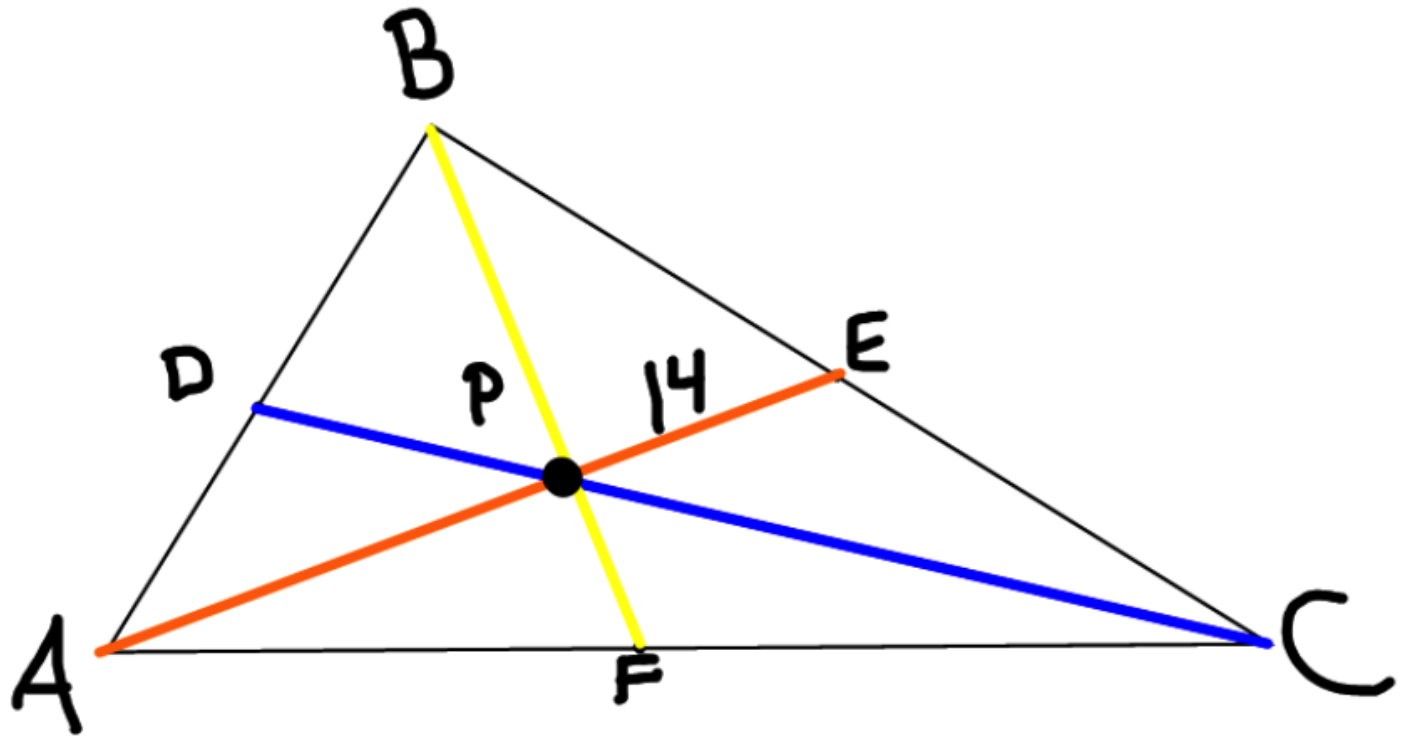


Given: $PE = 14$.

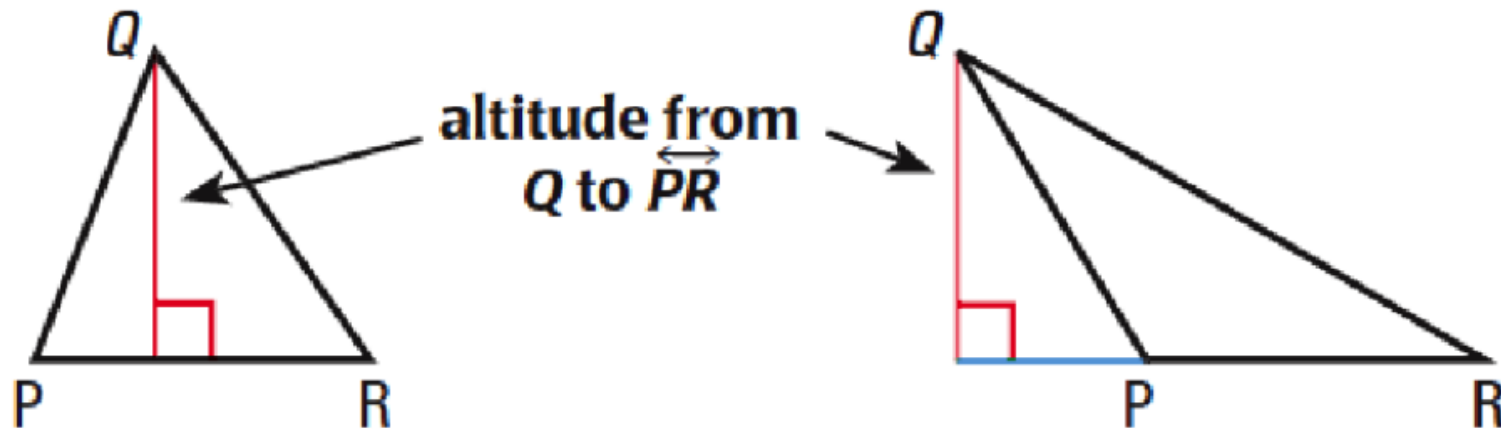
Find:

1. $AP =$

2. $AE =$



Altitude of a triangle- the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side



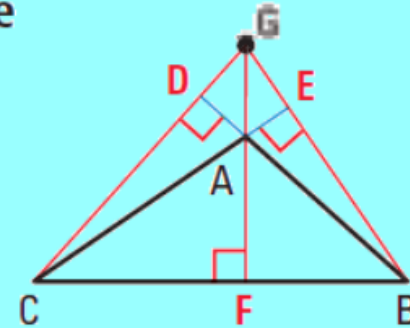
Orthocenter-

THEOREM*For Your Notebook***THEOREM 5.9** Concurrency of Altitudes of a Triangle

The lines containing the altitudes of a triangle are concurrent.

The lines containing \overline{AF} , \overline{BE} , and \overline{CD} meet at G .

Proof: Exs. 29–31, p. 323; p. 936



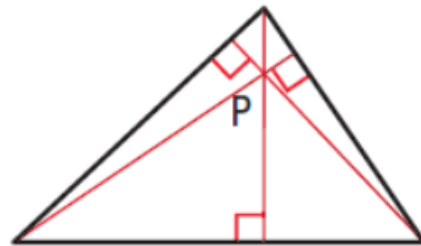
EXAMPLE 3 Find the orthocenter

Find the orthocenter P in an acute, a right, and an obtuse triangle.

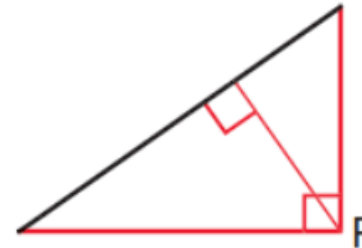
READ DIAGRAMS

The altitudes are shown in red. Notice that in the right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

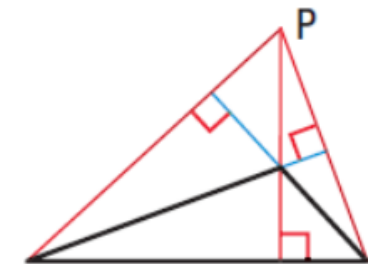
Solution



Acute triangle
 P is inside triangle.



Right triangle
 P is on triangle.



Obtuse triangle
 P is outside triangle.

ISOSCELES TRIANGLES In an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude from the vertex angle to the base are all the same segment. In an equilateral triangle, this is true for the special segment from any vertex.

Day 1
Assignment:
5.4 ws

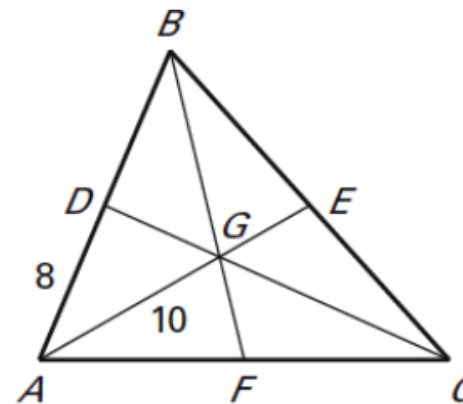
LESSON
5.4

Practice B

For use with pages 318–327

G is the centroid of $\triangle ABC$, $AD = 8$, $AG = 10$, and $CD = 18$. Find the length of the segment.

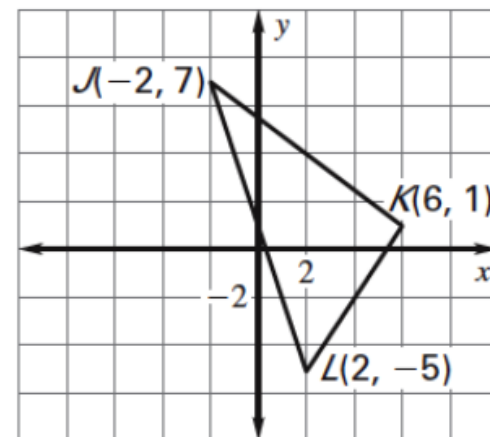
- | | |
|--------------------|--------------------|
| 1. \overline{BD} | 2. \overline{AB} |
| 3. \overline{EG} | 4. \overline{AE} |
| 5. \overline{CG} | 6. \overline{DG} |



21(21)

7. Use the graph shown.

- a.** Find the coordinates of M , the midpoint of \overline{JK} .
Use the median \overline{LM} to find the coordinates of the centroid P .
- b.** Find the coordinates of N , the midpoint of \overline{JL} .
Verify that $KP = \frac{2}{3}KN$.



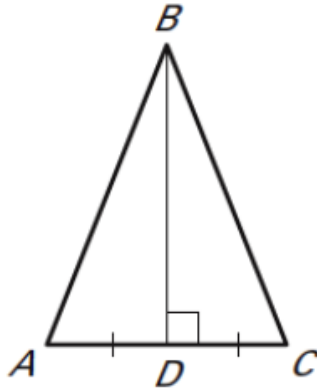
Find the coordinates of the centroid P of $\triangle ABC$.

8. $A(-7, -4), B(-3, 5), C(1, -4)$

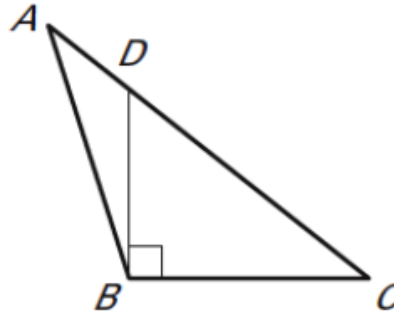
9. $A(0, -2), B(6, 1), C(9, -5)$

Is \overline{BD} a perpendicular bisector of $\triangle ABC$? Is \overline{BD} a median? an altitude?

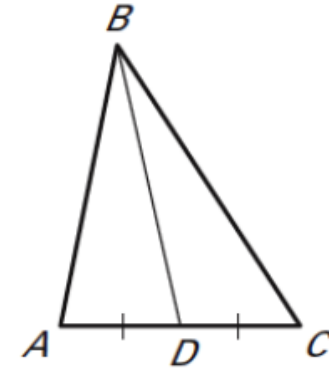
10.



11.

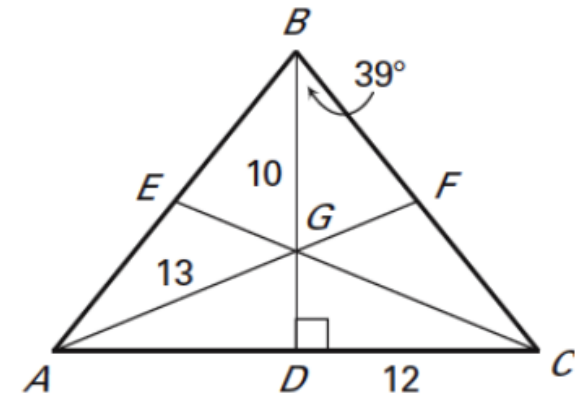


12.



Find the measurements.

- 13.** Given that $AB = BC$, find AD and $m\angle ABC$.
- 14.** Given that G is the centroid of $\triangle ABC$, find FG and BD .



LESSON
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Practice B *continued*
For use with pages 318–327

Copy and complete the statement for $\triangle HJK$ with medians \overline{HN} , \overline{JL} , and \overline{KM} , and centroid P .

15. $PN = \underline{\quad ? \quad} HN$

16. $PL = \underline{\quad ? \quad} JP$

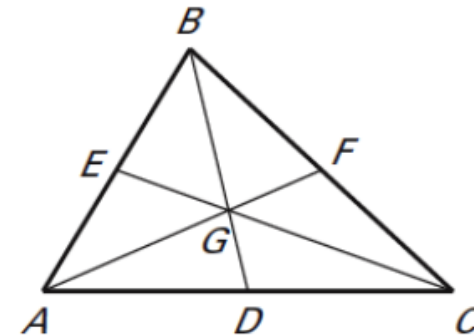
17. $KP = \underline{\quad ? \quad} KM$

Point G is the centroid of $\triangle ABC$. Use the given information to find the value of x .

18. $CG = 3x + 7$ and $CE = 6x$

19. $FG = x + 8$ and $AF = 9x - 6$

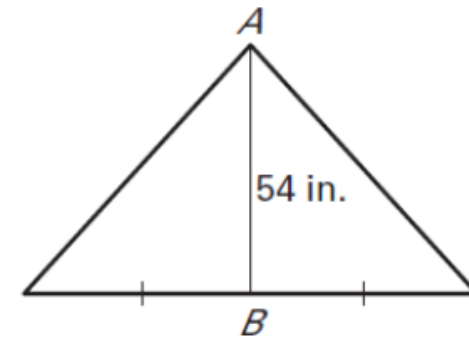
20. $BG = 5x - 1$ and $DG = 4x - 5$



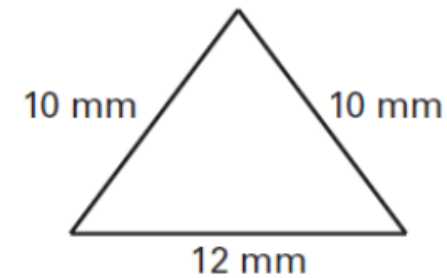
Complete the sentence with *always, sometimes, or never*.

- 21.** The median of a triangle is ___?___ the perpendicular bisector.
- 22.** The altitude of a triangle is ___?___ the perpendicular bisector.
- 23.** The medians of a triangle ___?___ intersect inside the triangle.
- 24.** The altitudes of a triangle ___?___ intersect inside the triangle.

- 25. House Decoration** You are going to put a decoration on your house in the triangular area above the front door. You want to place the decoration on the centroid of the triangle. You measure the distance from point A to point B (see figure). How far down from point A should you place the decoration? *Explain.*



- 26. Art Project** You are making an art piece which consists of different items of all shapes and sizes. You want to insert an isosceles triangle with the dimensions shown. In order for the triangle to fit, the height (altitude) must be less than 8.5 millimeters. Find the altitude. Will the triangle fit in your art piece?



Day 2 Assignment:

p. 322 (3-10, 13-22, 46-55,
skip # 8)