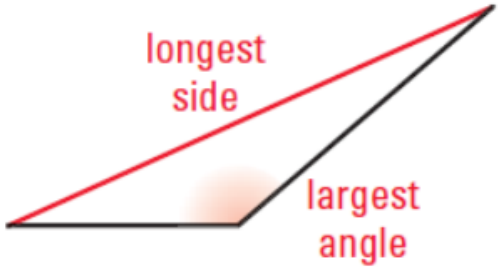
5.5 Use Inequalities in a Triangle

Goal • Find possible side lengths of a triangle.

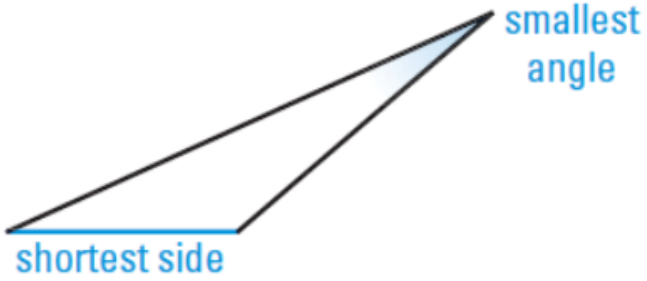
EXAMPLE 1 Relate side length and angle measure

Draw an obtuse scalene triangle. Find the largest angle and longest side and mark them in red. Find the smallest angle and shortest side and mark them in blue. What do you notice?

Solution



The longest side and largest angle are opposite each other.



The shortest side and smallest angle are opposite each other.

AVOID ERRORS

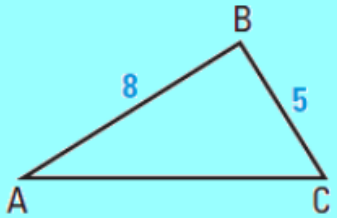
Be careful not to confuse the symbol $\angle$ meaning *angle* with the symbol $<$ meaning *is less than*. Notice that the bottom edge of the angle symbol is horizontal.

THEOREMS *For Your Notebook*

THEOREM 5.10

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

Proof: p. 329

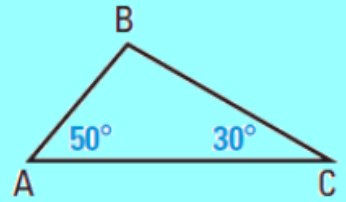


$AB > BC$, so $m\angle C > m\angle A$.

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

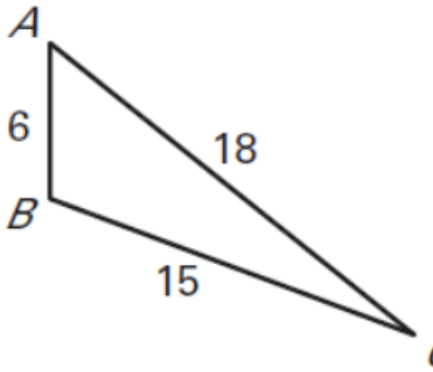
Proof: Ex. 24, p. 340



$m\angle A > m\angle C$, so $BC > AB$.

Name the smallest and largest angles of the triangle.

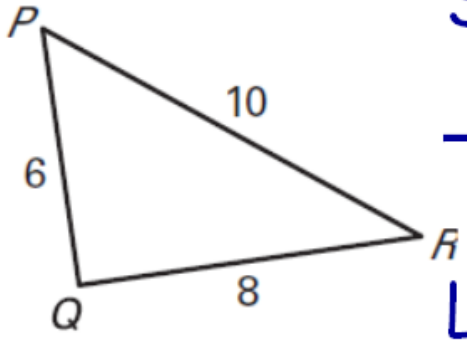
1.



Smallest: _____

Largest: _____

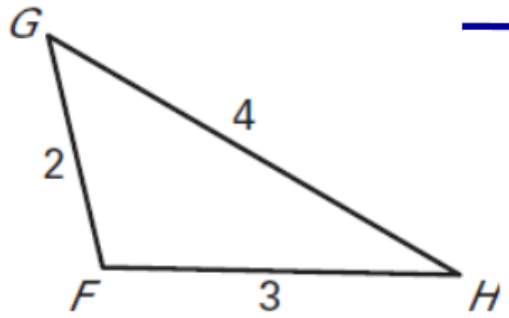
2.



Smallest: _____

Largest: _____

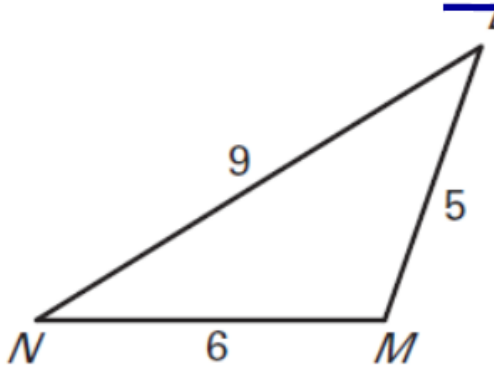
3.



Smallest: _____

Largest: _____

4.

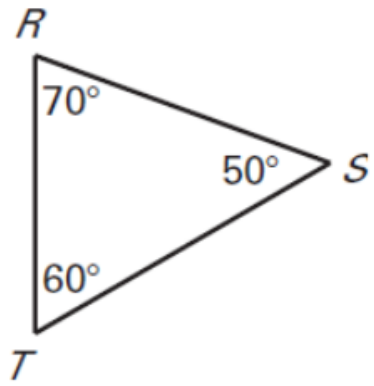


Smallest: _____

Largest: _____

Name the shortest and longest sides of the triangle.

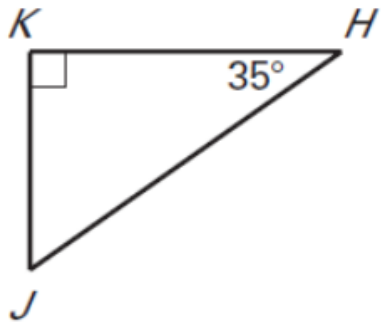
5.



Smallest: _____

Largest: _____

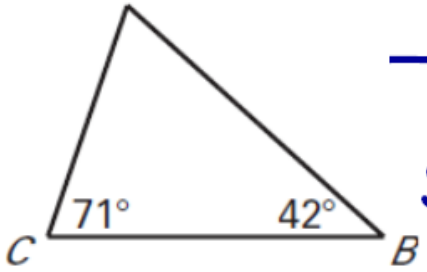
6.



Smallest: _____

Largest: _____

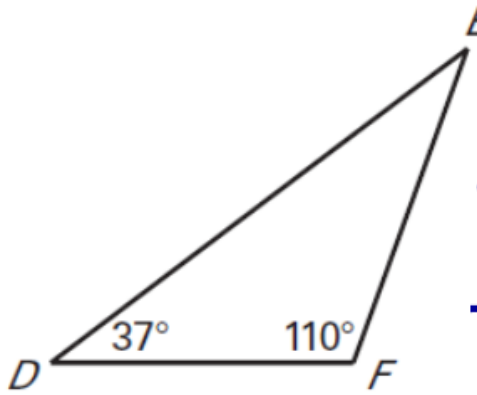
7.



Smallest: _____

Largest: _____

8.



Smallest: _____

Largest: _____

EXAMPLE 2 Standardized Test Practice

STAGE PROP You are constructing a stage prop that shows a large triangular mountain. The bottom edge of the mountain is about 27 feet long, the left slope is about 24 feet long, and the right slope is about 20 feet long. You are told that one of the angles is about 46° and one is about 59°. What is the angle measure of the peak of the mountain?

(A) 46° (B) 59° (C) 75° (D) 85°

Use the strips of paper with the given lengths to form a triangle (if possible). Record your data, then answer the questions.

Lengths	Will it form a triangle? (Yes/No)
4 in, 5 in, and 8 in	
2 in, 3 in, and 5 in	
3 in, 4 in, and 8 in	
2 in, 4 in, and 5 in	
4 in, 5 in, and 7 in	

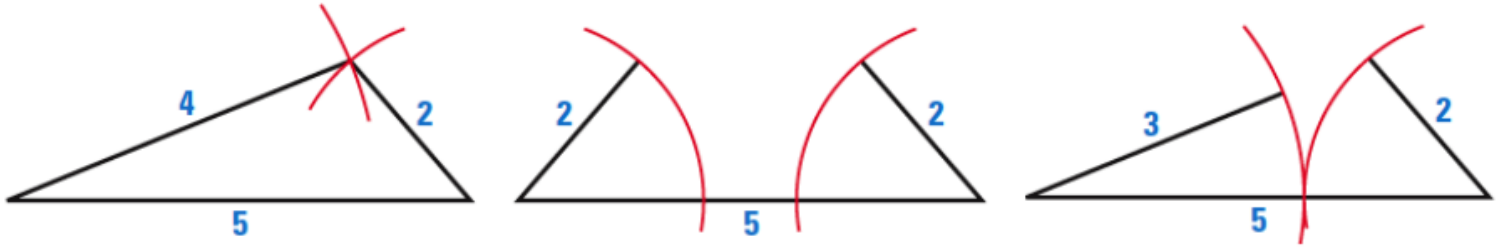
1. What relationship do you see in the lengths of the sides and whether it would form a triangle?

2. Write a rule that would apply to determining if any set of lengths would form a triangle.

What do you notice about the angles and the sides?

THE TRIANGLE INEQUALITY Not every group of three segments can be used to form a triangle. The lengths of the segments must fit a certain relationship.

For example, three attempted triangle constructions for sides with given lengths are shown below. Only the first set of side lengths forms a triangle.



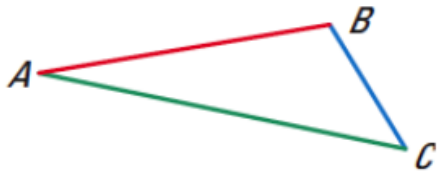
If you start with the longest side and attach the other two sides at its endpoints, you can see that the other two sides are not long enough to form a triangle in the second and third figures. This leads to the *Triangle Inequality Theorem*.

THEOREM

For Your Notebook

THEOREM 5.12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



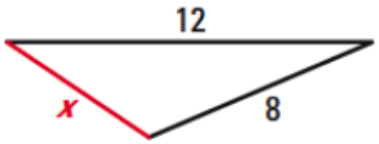
$AB + BC > AC$ $\quad AC + BC > AB$ $\quad AB + AC > BC$

Proof: Ex. 47, p. 334

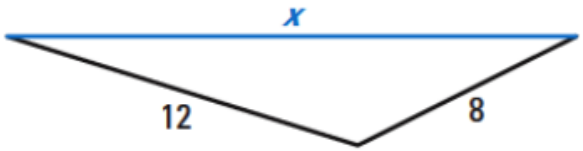
EXAMPLE 3 Find possible side lengths

ALGEBRA A triangle has one side of length 12 and another of length 8. Describe the possible lengths of the third side.

Small values of *x*



Large values of *x*



Assignment:

p. 331 (6-26 all)