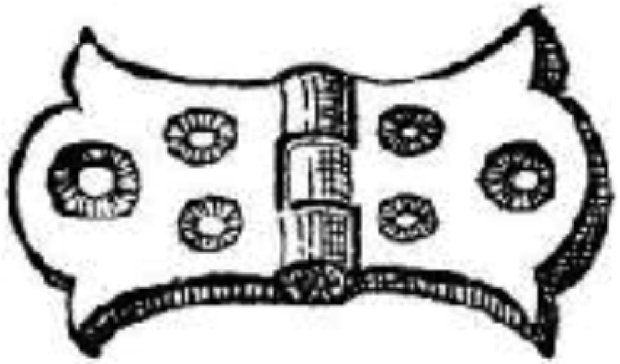
5.6 Inequalities in Two Triangles and Indirect Proof

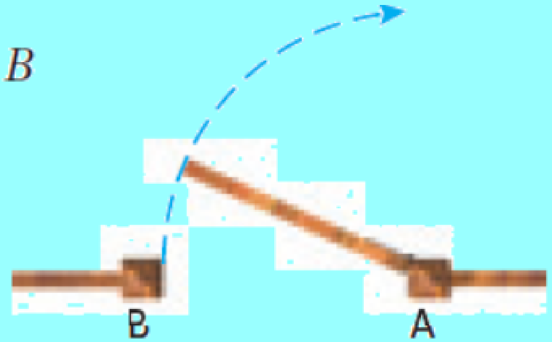
Goal • Use inequalities to make comparisons in two triangles.

*What mechanism makes a door open and close?

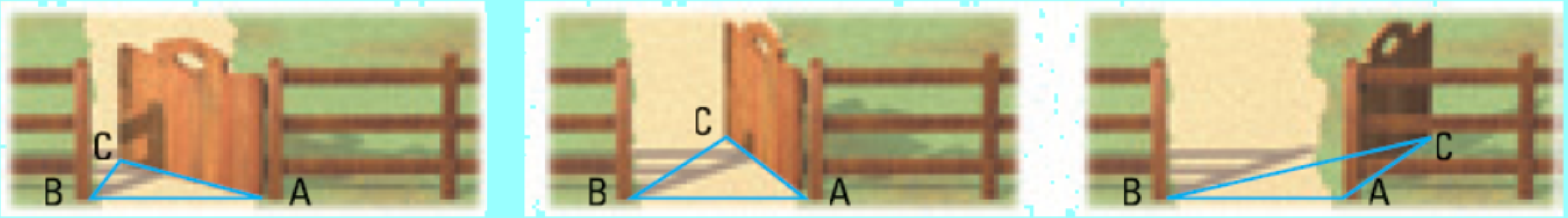
______ _



Imagine a gate between fence posts *A* and *B* that has hinges at *A* and swings open at *B*.



As the gate swings open, you can think of $\triangle ABC$, with side $\overline{AC}$ formed by the gate itself, side $\overline{AB}$ representing the distance between the fence posts, and side $\overline{BC}$ representing the opening between post *B* and the outer edge of the gate.

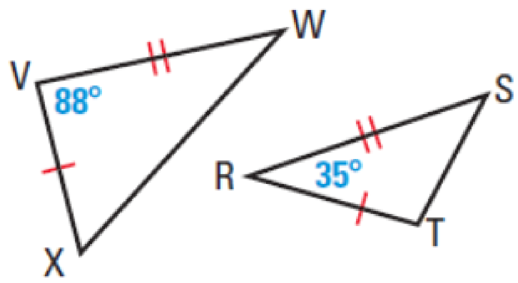


Notice that as the gate opens wider, both the measure of $\angle A$ and the distance *CB* increase. This suggests the *Hinge Theorem*.

THEOREMS — *For Your Notebook*

THEOREM 5.13 Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.

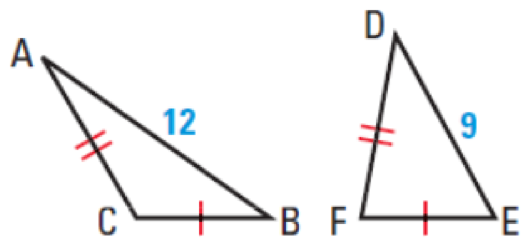


WX > ST

Proof: Ex. 28, p. 341

THEOREM 5.14 Converse of the Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

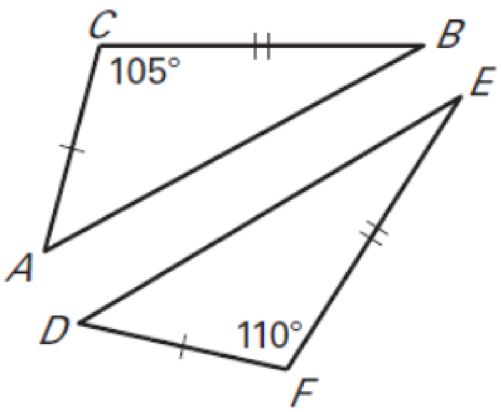


$m\angle C > m\angle F$

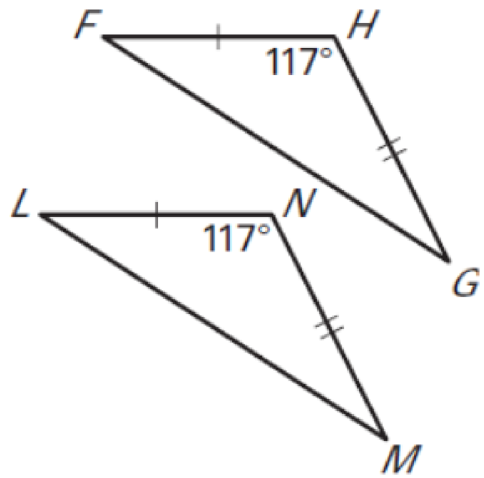
Proof: Example 4, p. 338

Complete with <, >, or = .

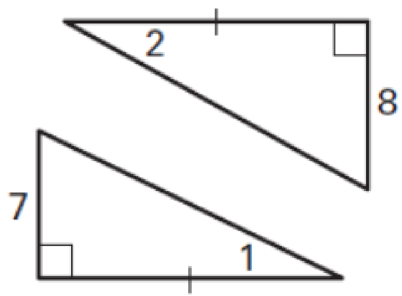
1. AB __?__ DE



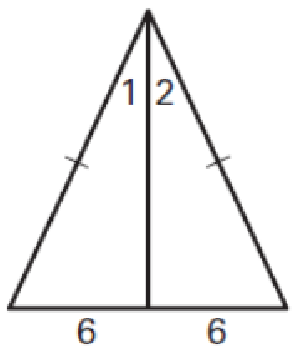
2. FG __?__ LM



3. $m\angle 1$ __?__ $m\angle 2$

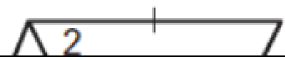


4. $m\angle 1$ __?__ $m\angle 2$



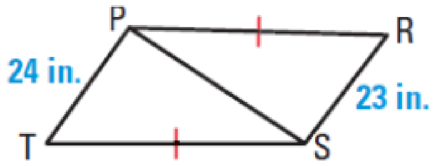
5. MS __?__ LS

6. $m\angle 1$ __?__ $m\angle 2$



EXAMPLE 1 Use the Converse of the Hinge Theorem

Given that $\overline{ST} \cong \overline{PR}$, how does $\angle PST$ compare to $\angle SPR$?



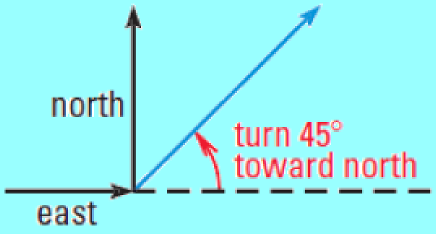
Solution

You are given that $\overline{ST} \cong \overline{PR}$ and you know that $\overline{PS} \cong \overline{PS}$ by the Reflexive Property. Because 24 inches > 23 inches, $PT > RS$. So, two sides of $\triangle STP$ are congruent to two sides of $\triangle PRS$ and the third side in $\triangle STP$ is longer.

▶ By the Converse of the Hinge Theorem, $m\angle PST > m\angle SPR$.

EXAMPLE 2 Solve a multi-step problem

BIKING Two groups of bikers leave the same camp heading in opposite directions. Each group goes 2 miles, then changes direction and goes 1.2 miles. Group A starts due east and then turns 45° toward north as shown. Group B starts due west and then turns 30° toward south.



Which group is farther from camp? Explain your reasoning.

Solution

Draw a diagram and mark the given measures. The distances biked and the distances back to camp form two triangles, with congruent 2 mile sides and congruent 1.2 mile sides. Add the third sides of the triangles to your diagram.

Next use linear pairs to find and mark the included angles of 150° and 135°.

▶ Because 150° > 135°, Group B is farther from camp by the Hinge Theorem.

Indirect Reasoning

Indirect Proof-In an indirect proof you start by making an assumption that the desired conclusion is false

By then showing that this assumption leads to a logical impossibility, you prove the original statement true by contradiction.

Everything that we have done so far was by direct proof. We proved that something was TRUE. With indirect proof we prove that something is FALSE.

KEY CONCEPT *For Your Notebook*

How to Write an Indirect Proof

STEP 1 **Identify** the statement you want to prove. **Assume** temporarily that this statement is false by assuming that its opposite is true.

STEP 2 **Reason** logically until you reach a contradiction.

STEP 3 **Point out** that the desired conclusion must be true because the contradiction proves the temporary assumption false.

INDIRECT REASONING Suppose a student looks around the cafeteria, concludes that hamburgers are not being served, and explains as follows.

> *At first I assumed that we are having hamburgers because today is Tuesday and Tuesday is usually hamburger day.*
>
> *There is always ketchup on the table when we have hamburgers, so I looked for the ketchup, but I didn't see any.*
>
> *So, my assumption that we are having hamburgers must be false.*

EXAMPLE 3 Write an indirect proof

Write an indirect proof that an odd number is not divisible by 4.

GIVEN ▶ x is an odd number.

PROVE ▶ x is not divisible by 4.

Solution

STEP 1 Assume temporarily that x is divisible by 4. This means that $\frac{x}{4} = n$ for some whole number n. So, multiplying both sides by 4 gives $x = 4n$.

STEP 2 If x is odd, then, by definition, x cannot be divided evenly by 2. However, $x = 4n$ so $\frac{x}{2} = \frac{4n}{2} = 2n$. We know that $2n$ is a whole number because n is a whole number, so x *can* be divided evenly by 2. This contradicts the given statement that x is odd.

STEP 3 Therefore, the assumption that x is divisible by 4 must be false, which proves that x is not divisible by 4.

READ VOCABULARY

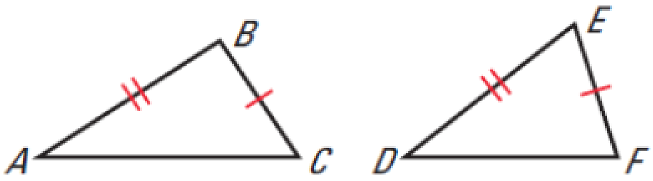
You have reached a *contradiction* when you have two statements that cannot both be true at the same time.

EXAMPLE 4 Prove the Converse of the Hinge Theorem

Write an indirect proof of Theorem 5.14.

GIVEN ▶ $\overline{AB} \cong \overline{DE}$
$\overline{BC} \cong \overline{EF}$
$AC > DF$

PROVE ▶ $m\angle B > m\angle E$



Proof Assume temporarily that $m\angle B \ngtr m\angle E$. Then, it follows that either $m\angle B = m\angle E$ or $m\angle B < m\angle E$.

Case 1 If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate and $AC = DF$.

Case 2 If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem.

Both conclusions contradict the given statement that $AC > DF$. So, the temporary assumption that $m\angle B \ngtr m\angle E$ cannot be true. This proves that $m\angle B > m\angle E$.

Day 1

Assignment:

5.6 ws

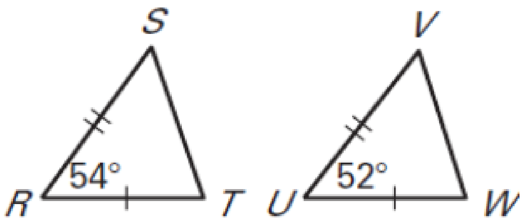
LESSON 5.6

Practice B

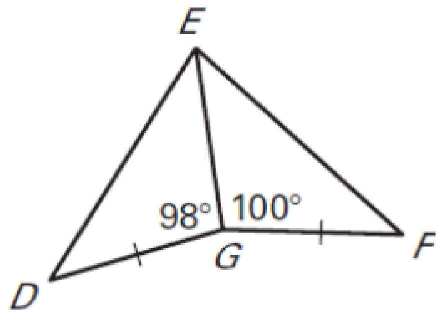
For use with pages 335–341

Complete with <, >, or = . *Explain.*

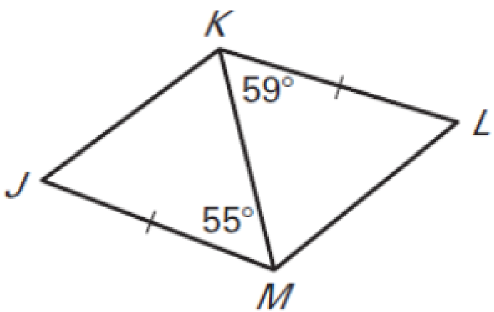
1. ST __?__ VW



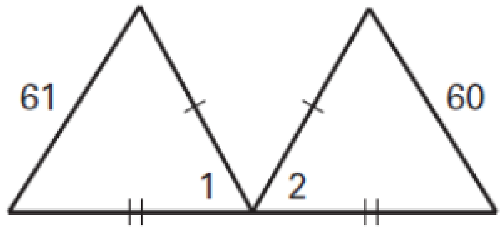
2. DE __?__ EF



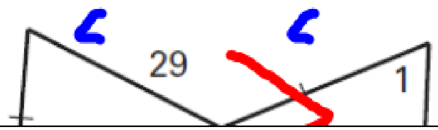
3. JK __?__ LM



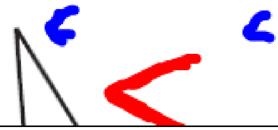
4. $m\angle 1$ __?__ $m\angle 2$



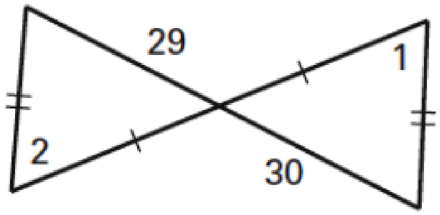
5. m∉ 1 __?__ m∉ 2



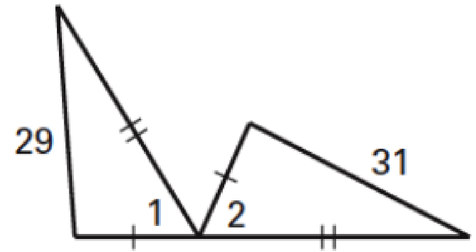
6. m∉ 1 __?__ m∉ 2



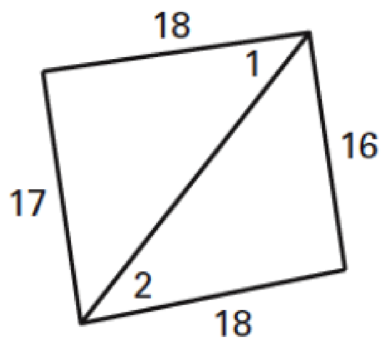
5. $m\angle 1$ _?_ $m\angle 2$



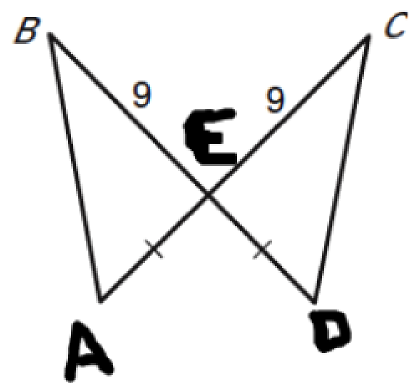
6. $m\angle 1$ _?_ $m\angle 2$



7. $m\angle 1$ _?_ $m\angle 2$

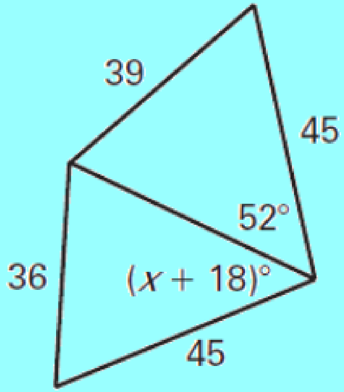


8. AB _?_ CD

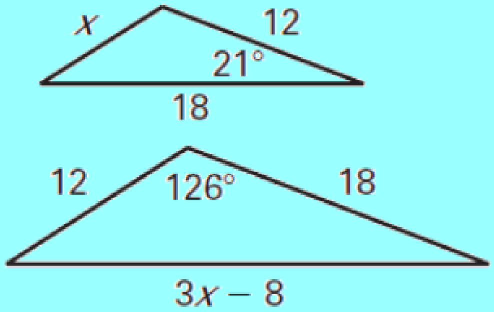


Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of *x*.

9.



10.

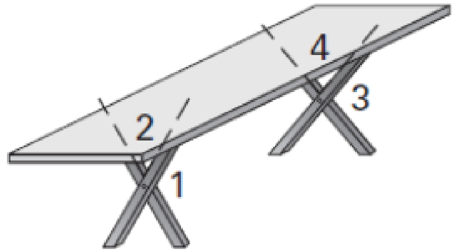


Write a temporary assumption you could make to prove the conclusion indirectly.

11. If two lines in a plane are parallel, then the two lines do not contain two sides of a triangle.

12. If two parallel lines are cut by a transversal so that a pair of consecutive interior angles is congruent, then the transversal is perpendicular to the parallel lines.

13. Table Making All four legs of the table shown have identical measurements, but they are attached to the table top so that ∠3 is smaller than ∠1.



a. Use the Hinge Theorem to *explain* why the table top is not level.

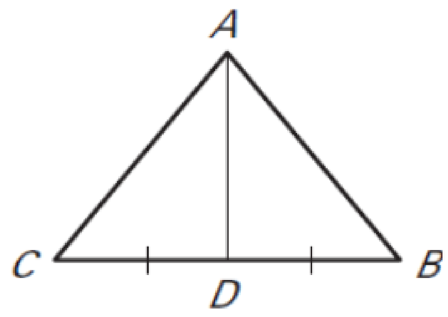
b. Use the Converse of the Hinge Theorem to *explain* how to use a length measure to determine when ∠4 ≅ ∠2 in reattaching the rear pair of legs to make the table level.

14. Fishing Contest One contestant in a catch-and-release fishing contest spends the morning at a location 1.8 miles due north of the starting point, then goes 1.2 miles due east for the rest of the day. A second contestant starts out 1.2 miles due east of the starting point, then goes another 1.8 miles in a direction 84° south of due east to spend the rest of the day. Which angler is farther from the starting point at the end of the day? *Explain* how you know.

15. **Indirect Proof** Arrange statements A–F in order to write an indirect proof of Case 1.

GIVEN: $\overline{AD}$ is a median of $\triangle ABC$.
$\angle ADB \cong \angle ADC$

PROVE: $AB = AC$



Case 1:

A. Then $m\angle ADB < m\angle ADC$ by the converse of the Hinge Theorem.

B. Then $\overline{BD} \cong \overline{CD}$ by the definition of midpoint. Also, $\overline{AD} \cong \overline{AD}$ by the reflexive property.

C. This contradiction shows that the temporary assumption that $AB < AC$ is false.

D. But this contradicts the given statement that $\angle ADB \cong \angle ADC$.

E. Because $\overline{AD}$ is a median of $\triangle ABC$, D is the midpoint of $\overline{BC}$.

F. Temporarily assume that $AB < AC$.

16. **Indirect Proof** There are two cases to consider for the proof in Exercise 15. Write an indirect proof for Case 2.

Answer Key

Lesson 5.6

Practice Level B

1. >; Hinge Thm. with $m\angle R > m\angle U$

2. <; Hinge Thm. with $m\angle DGE < m\angle EGF$

3. <; Hinge Thm. with $m\angle JMK < m\angle LKM$

4. >; Converse of Hinge Thm. with the side opposite $\angle 1$ longer than the side opposite $\angle 2$.

5. >; Converse of Hinge Thm. with the side opposite $\angle 1$ longer than the side opposite $\angle 2$.

6. <; Converse of Hinge Thm. with the side opposite $\angle 1$ shorter than the side opposite $\angle 2$.

7. >; Converse of Hinge Thm. with the side opposite $\angle 1$ longer than the side opposite $\angle 2$.

8. =; The triangles are $\cong$ by SAS. **9.** $x < 34$

10. $x > 4$ **11.** Assume temporarily that the two parallel lines contain two sides of a triangle.

12. Assume temporarily that the transversal is not perpendicular to the parallel lines.

13. a. Because $m\angle 3 < m\angle 1$, by the Hinge Thm, the far side of the table is lower than the near side. **b.** By the Converse of the Hinge Thm., $\angle 4$ will
be larger than $\angle 2$ until the distance between the tops of each pair of legs is the same.

14. the second angler; The included $\angle$ for the

Day 2 Assignment:

p. 338 (3-9, 16-18, 22,
29-35)