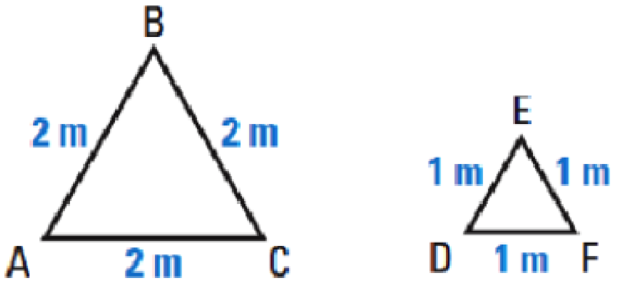
6.1 Ratios, Proportions, and the Geometric Mean

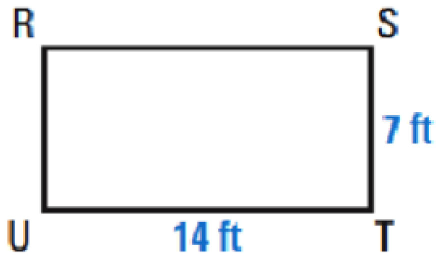
Goal • Solve problems by writing and solving proportions.

If a and b are two numbers or quantities and $b \neq 0$, then the **ratio of a to b** is $\frac{a}{b}$. The ratio of a to b can also be written as $a:b$.

For example, the ratio of a side length in $\triangle ABC$ to a side length in $\triangle DEF$ can be written as $\frac{2}{1}$ or $2:1$.



Ratios are usually expressed in simplest form. Two ratios that have the same simplified form are called *equivalent ratios*. The ratios 7 : 14 and 1 : 2 in the example below are *equivalent*.



$$\frac{\text{width of } RSTU}{\text{length of } RSTU} = \frac{7 \text{ ft}}{14 \text{ ft}} = \frac{1}{2}$$

EXAMPLE 1 Simplify ratios

Simplify the ratio.

a. 64 m : 6 m

b. $\frac{5 \text{ ft}}{20 \text{ in.}}$

Solution

a. Write 64 m : 6 m as $\frac{64 \text{ m}}{6 \text{ m}}$. Then divide out the units and simplify.

$$\frac{64 \cancel{\text{m}}}{6 \cancel{\text{m}}} = \frac{32}{3} = 32:3$$

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{5 \text{ ft}}{20 \text{ in.}} = \frac{5 \cancel{\text{ft}}}{20 \cancel{\text{in.}}} \cdot \frac{12 \cancel{\text{in.}}}{1 \cancel{\text{ft}}} = \frac{60}{20} = \frac{3}{1}$$

GUIDED PRACTICE for Example 1

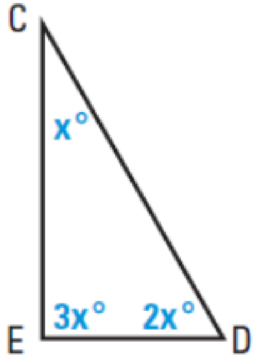
Simplify the ratio.

1. 24 yards to 3 yards

2. 150 cm : 6 m

EXAMPLE 3 Use extended ratios

ALGEBRA The measures of the angles in $\triangle CDE$ are in the *extended ratio* of 1:2:3. Find the measures of the angles.



Solution

Begin by sketching the triangle. Then use the extended ratio of 1 : 2 : 3 to label the measures as $x°$, $2x°$, and $3x°$.

$x° + 2x° + 3x° = 180°$ — Triangle Sum Theorem

$6x = 180$ — Combine like terms.

$x = 30$ — Divide each side by 6.

▶ The angle measures are 30°, 2(30°) = 60°, and 3(30°) = 90°.

The measures of the angles of a triangle are in the extended ratio given. Find the measures of the angles of the triangle.

16. $1:1:1$

17. $1:1:2$

18. $2:3:4$

PROPORTIONS An equation that states that two ratios are equal is called a **proportion**.

extreme → a ... c ← mean

$$\frac{a}{b} = \frac{c}{d}$$

mean → b ... d ← extreme

The numbers b and c are the **means** of the proportion. The numbers a and d are the **extremes** of the proportion.

The property below can be used to solve proportions. To *solve a proportion*, you find the value of any variable in the proportion.

KEY CONCEPT *For Your Notebook*

A Property of Proportions

1. Cross Products Property In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

$$\frac{2}{3} = \frac{4}{6} \qquad 3 \cdot 4 = 12 \qquad 2 \cdot 6 = 12$$

EXAMPLE 4 Solve proportions

xy ALGEBRA **Solve the proportion.**

a. $\frac{5}{10} = \frac{x}{16}$

b. $\frac{1}{y+1} = \frac{2}{3y}$

Solution

a. $\frac{5}{10} = \frac{x}{16}$ Write original proportion.

$5 \cdot 16 = 10 \cdot x$ Cross Products Property

$80 = 10x$ Multiply.

$8 = x$ Divide each side by 10.

b. $\frac{1}{y+1} = \frac{2}{3y}$ Write original proportion.

$1 \cdot 3y = 2(y + 1)$ Cross Products Property

$3y = 2y + 2$ Distributive Property

$y = 2$ Subtract $2y$ from each side.

ANOTHER WAY

In part (a), you could multiply each side by the denominator, 16. Then $16 \cdot \frac{5}{10} = 16 \cdot \frac{x}{16}$, so $8 = x$.

Solve the proportion.

5. $\frac{2}{x} = \frac{5}{8}$

6. $\frac{1}{x-3} = \frac{4}{3x}$

7. $\frac{y-3}{7} = \frac{y}{14}$

EXAMPLE 5 Solve a real-world problem

SCIENCE As part of an environmental study, you need to estimate the number of trees in a 150 acre area. You count 270 trees in a 2 acre area and you notice that the trees seem to be evenly distributed. Estimate the total number of trees.

KEY CONCEPT *For Your Notebook*

Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

EXAMPLE 6 Find a geometric mean

Find the geometric mean of 24 and 48.

$\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Solution

$x = \sqrt{ab}$ — Definition of geometric mean

$= \sqrt{24 \cdot 48}$ — Substitute 24 for *a* and 48 for *b*.

$= \sqrt{24 \cdot 24 \cdot 2}$ — Factor.

$= 24\sqrt{2}$ — Simplify.

▶ The geometric mean of 24 and 48 is $24\sqrt{2} \approx 33.9$.

$\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Find the geometric mean of the two numbers.

9. 12 and 27

10. 18 and 54

11. 16 and 18

Day 2 Assignment:

p. 360 (3-45 mult. of 3)