6.2 Use Proportions to Solve Geometry Problems

Goal • Use proportions to solve geometry problems.

In lesson 6.1 we learned to use the Cross Products Property to write equations that are equivalent to a given proportion.

Lesson 6.2 introduces us to three more ways to solve these problems.

KEY CONCEPT *For Your Notebook*

Additional Properties of Proportions

2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

3. If you interchange the means of a proportion, then you form another true proportion.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

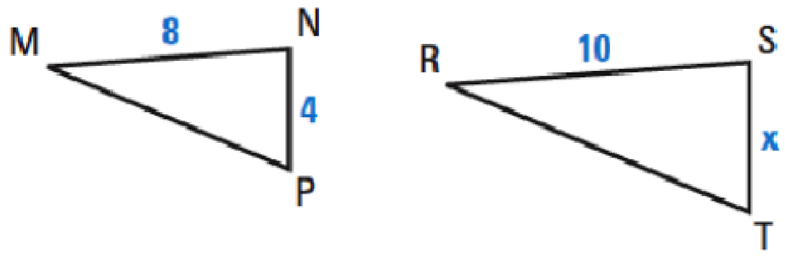
4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Know these relationships!!

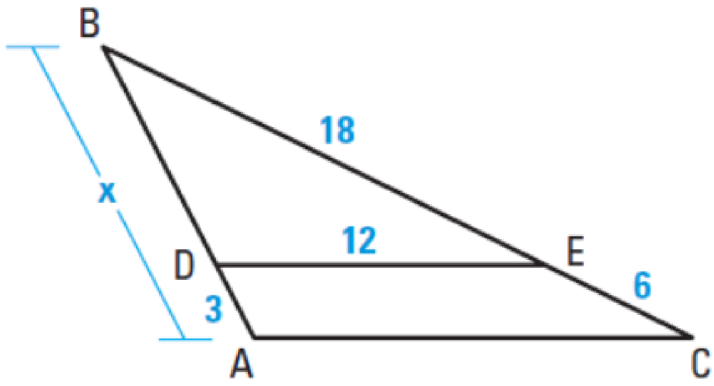
EXAMPLE 1 Use properties of proportions

In the diagram, $\frac{MN}{RS} = \frac{NP}{ST}$.
Write four true proportions.



EXAMPLE 2 Use proportions with geometric figures

ALGEBRA In the diagram, $\frac{BD}{DA} = \frac{BE}{EC}$. Find BA and BD.



Solution

$$\frac{BD}{DA} = \frac{BE}{EC}$$ Given

$$\frac{BD + DA}{DA} = \frac{BE + EC}{EC}$$ Property of Proportions (Property 4)

$$\frac{x}{3} = \frac{18 + 6}{6}$$ Substitution Property of Equality

$$6x = 3(18 + 6)$$ Cross Products Property

$$x = 12$$ Solve for *x*.

▸ So, $BA = 12$ and $BD = 12 - 3 = 9$.

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Scale drawing-A drawing that is the same shape as the object it represents

Scale-A ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object

In art class you may have made a scale drawing. Model cars are scale figures of the original. Blue prints are scale drawings of the actual building.

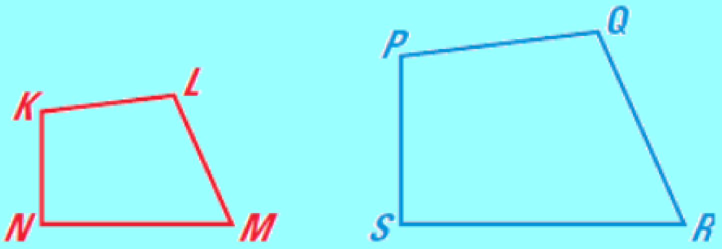
A map is a scale of the actual state or country

Perimeters- the ratio of lengths in similar polygons is the same as the scale factor

THEOREM *For Your Notebook*

THEOREM 6.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then $\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}$.

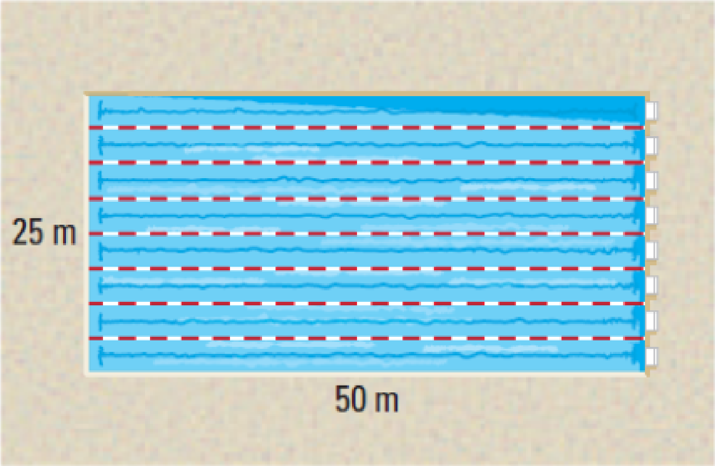
Proof: Ex. 38, p. 379

EXAMPLE 4 Find perimeters of similar figures

SWIMMING A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.

a. Find the scale factor of the new pool to an Olympic pool.

b. Find the perimeter of an Olympic pool and the new pool.



Solution

a. Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$.

b. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. You can use Theorem 6.1 to find the perimeter x of the new pool.

$\frac{x}{150} = \frac{4}{5}$ **Use Theorem 6.1 to write a proportion.**

$x = 120$ **Multiply each side by 150 and simplify.**

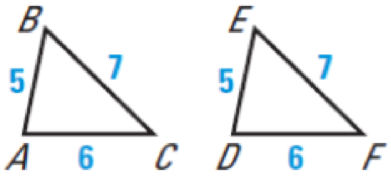
▶ The perimeter of the new pool is 120 meters.

ANOTHER WAY

Another way to solve Example 4 is to write the scale factor as the decimal 0.8. Then, multiply the perimeter of the Olympic pool by the scale factor to get the perimeter of the new pool:

$0.8(150) = 120.$

SIMILARITY AND CONGRUENCE Notice that any two congruent figures are also similar. Their scale factor is 1 : 1. In $\triangle ABC$ and $\triangle DEF$, the scale factor is $\frac{5}{5} = 1$. You can write $\triangle ABC \sim \triangle DEF$ and $\triangle ABC \cong \triangle DEF$.



READ VOCABULARY
For example, *corresponding lengths* in similar triangles include side lengths, altitudes, medians, midsegments, and so on.

CORRESPONDING LENGTHS You know that perimeters of similar polygons are in the same ratio as corresponding side lengths. You can extend this concept to other segments in polygons.

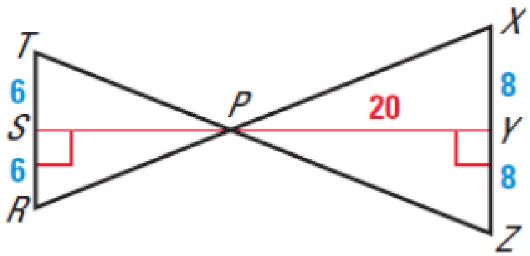
KEY CONCEPT *For Your Notebook*

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 5 Use a scale factor

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



Solution

First, find the scale factor of $\triangle TPR$ to $\triangle XPZ$.

$$\frac{TR}{XZ} = \frac{6 + 6}{8 + 8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$$\frac{PS}{PY} = \frac{3}{4}$$ **Write proportion.**

$$\frac{PS}{20} = \frac{3}{4}$$ **Substitute 20 for *PY*.**

$$PS = 15$$ **Multiply each side by 20 and simplify.**

▶ The length of the altitude $\overline{PS}$ is 15.

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✓ GUIDED PRACTICE for Example 5

7. In the diagram, $\triangle JKL \sim \triangle EFG$. Find the length of the median $\overline{KM}$.

Assignment:

6.3 Wkst

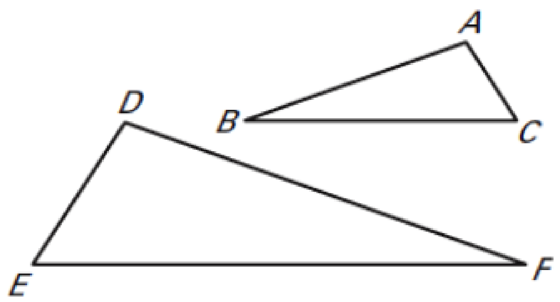
LESSON 6.3

Practice B

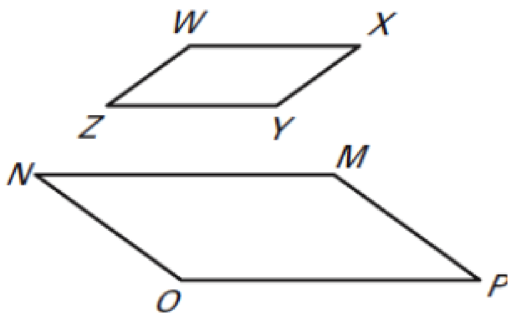
For use with pages 371–379

List all pairs of congruent angles for the figures. Then write the ratios of the corresponding sides in a statement of proportionality.

1. $\triangle ABC \sim \triangle DFE$



2. $WXYZ \sim MNOP$

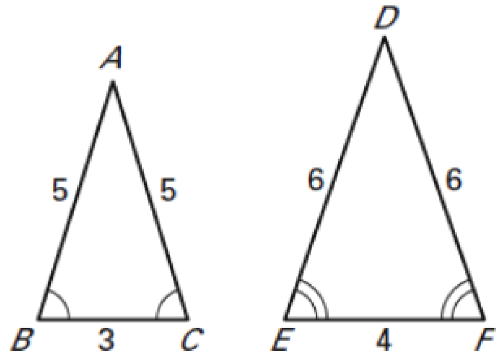


3. Multiple Choice Triangles ABC and DEF are similar. Which statement is not correct?

A. $\frac{AB}{DE} = \frac{BC}{EF}$ **B.** $\frac{CA}{FD} = \frac{AB}{DE}$ **C.** $\angle A \cong \angle F$

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

4.



5.