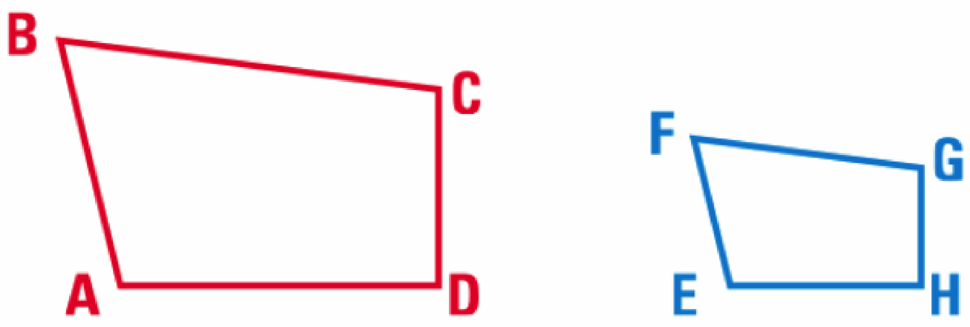
6.3 Use Similar Polygons

Goal • Use proportions to identify similar polygons.

Similar Polygons

two polygons which have
CONGRUENT corresponding angles
and
proportional corresponding side lengths



$ABCD \sim EFGH$

(Symbol for similarity)

Corresponding angles

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$

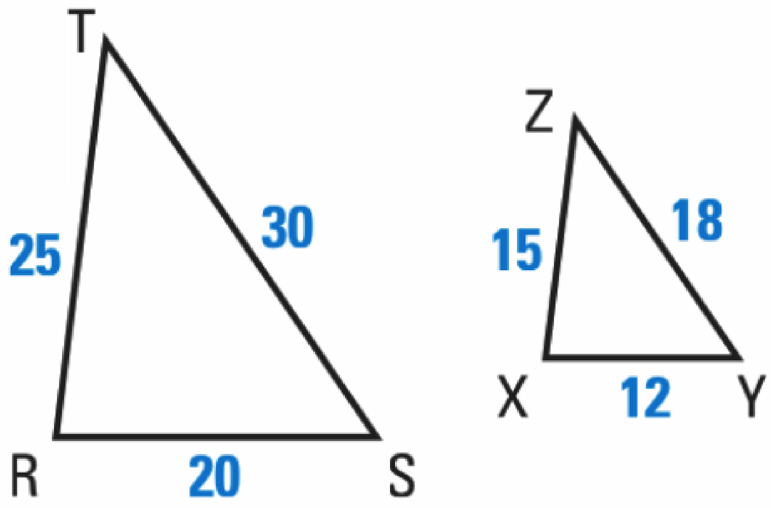
Ratios of corresponding sides

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

EXAMPLE 1 Use similarity statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

a. List all pairs of congruent angles.



b. Check that the ratios of corresponding side lengths are equal.

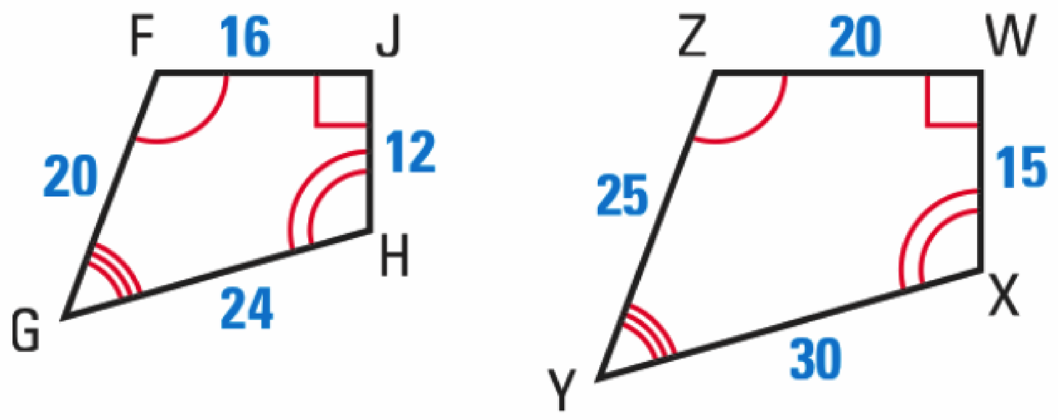
c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.

READ VOCABULARY

In a *statement of proportionality*, any pair of ratios forms a true proportion.

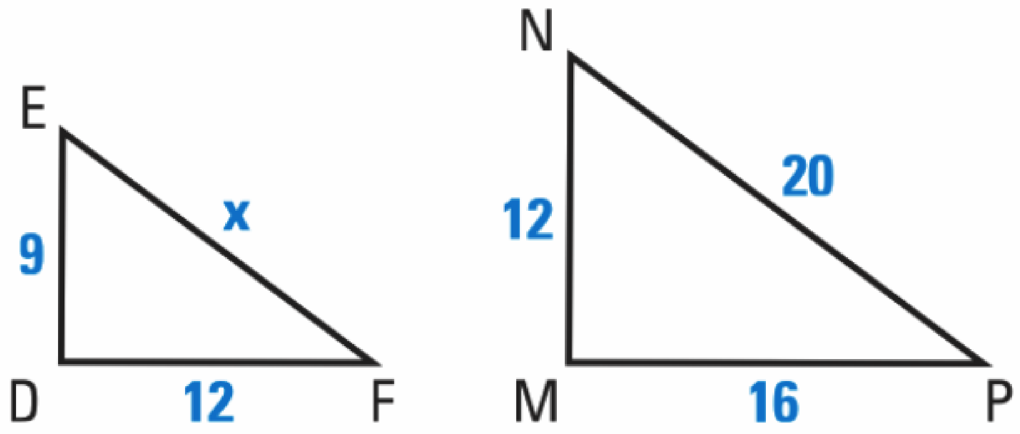
EXAMPLE 2 Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ZYXW* to *FGHJ*.



EXAMPLE 3 Use similar polygons

ALGEBRA In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x.

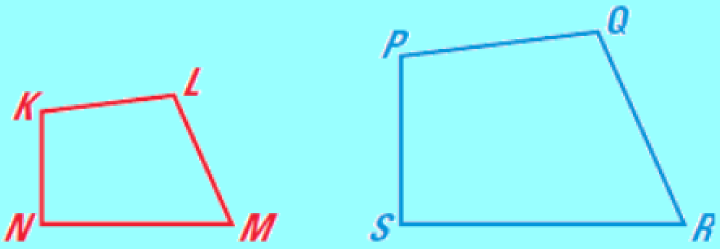


Perimeters- the ratio of lengths in similar polygons is the same as the scale factor

THEOREM — *For Your Notebook*

THEOREM 6.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

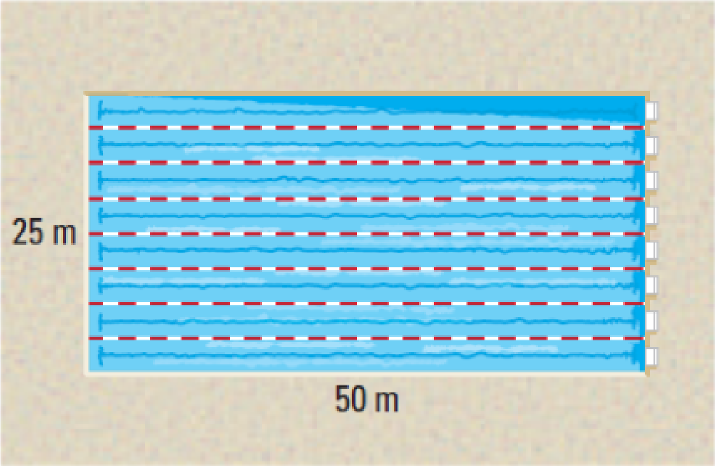


If $KLMN \sim PQRS$, then $\dfrac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \dfrac{KL}{PQ} = \dfrac{LM}{QR} = \dfrac{MN}{RS} = \dfrac{NK}{SP}$.

Proof: Ex. 38, p. 379

EXAMPLE 4 Find perimeters of similar figures

SWIMMING A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.



a. Find the scale factor of the new pool to an Olympic pool.

b. Find the perimeter of an Olympic pool and the new pool.

Solution

a. Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$.

b. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. You can use Theorem 6.1 to find the perimeter x of the new pool.

$\frac{x}{150} = \frac{4}{5}$ **Use Theorem 6.1 to write a proportion.**

$x = 120$ **Multiply each side by 150 and simplify.**

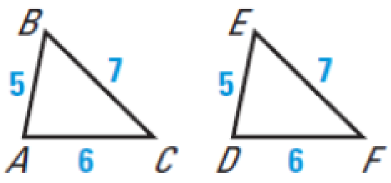
▶ The perimeter of the new pool is 120 meters.

ANOTHER WAY

Another way to solve Example 4 is to write the scale factor as the decimal 0.8. Then, multiply the perimeter of the Olympic pool by the scale factor to get the perimeter of the new pool:

$0.8(150) = 120.$

SIMILARITY AND CONGRUENCE Notice that any two congruent figures are also similar. Their scale factor is 1 : 1. In $\triangle ABC$ and $\triangle DEF$, the scale factor is $\frac{5}{5} = 1$. You can write $\triangle ABC \sim \triangle DEF$ and $\triangle ABC \cong \triangle DEF$.



READ VOCABULARY
For example, *corresponding lengths* in similar triangles include side lengths, altitudes, medians, midsegments, and so on.

CORRESPONDING LENGTHS You know that perimeters of similar polygons are in the same ratio as corresponding side lengths. You can extend this concept to other segments in polygons.

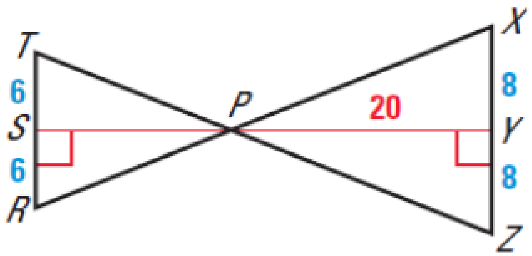
KEY CONCEPT *For Your Notebook*

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 5 Use a scale factor

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



Solution

First, find the scale factor of $\triangle TPR$ to $\triangle XPZ$.

$$\frac{TR}{XZ} = \frac{6+6}{8+8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ **Write proportion.**

$\frac{PS}{20} = \frac{3}{4}$ **Substitute 20 for *PY*.**

$PS = 15$ **Multiply each side by 20 and simplify.**

▶ The length of the altitude $\overline{PS}$ is 15.

AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Example 5

7. In the diagram, $\triangle JKL \sim \triangle EFG$. Find the length of the median $\overline{KM}$.

Assignment:

6.3 Wkst

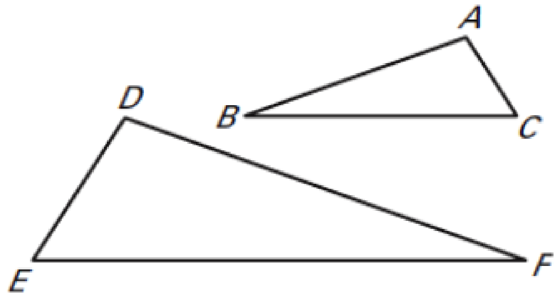
LESSON 6.3

Practice B

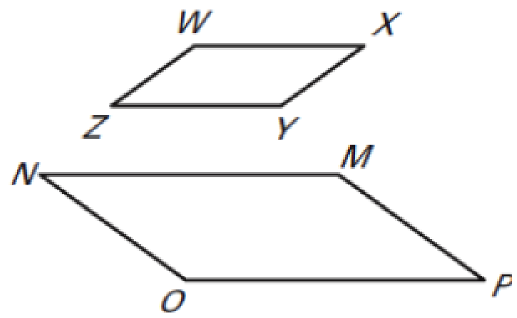
For use with pages 371–379

List all pairs of congruent angles for the figures. Then write the ratios of the corresponding sides in a statement of proportionality.

1. $\triangle ABC \sim \triangle DFE$



2. $WXYZ \sim MNOP$

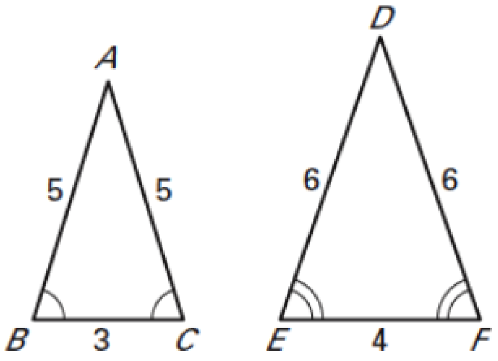


3. Multiple Choice Triangles ABC and DEF are similar. Which statement is not correct?

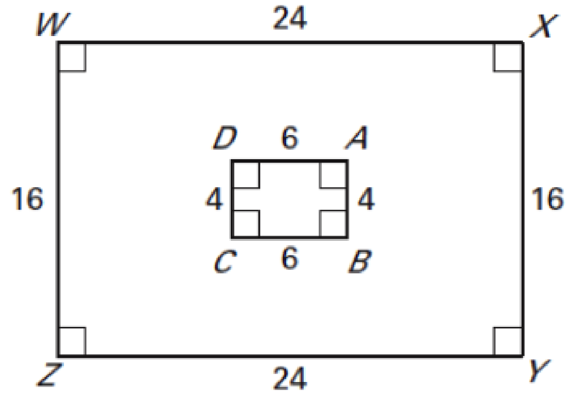
A. $\frac{AB}{DE} = \frac{BC}{EF}$ **B.** $\frac{CA}{FD} = \frac{AB}{DE}$ **C.** $\angle A \cong \angle F$

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

4.



5.



In the diagram, *WXYZ* ~ *MNOP*.

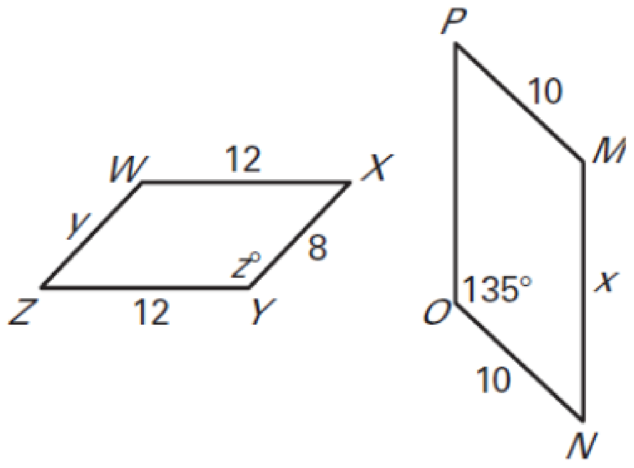
6. Find the scale factor of *WXYZ* to *MNOP*.

7. Find the values of x, y, and z.

8. Find the perimeter of *WXYZ*.

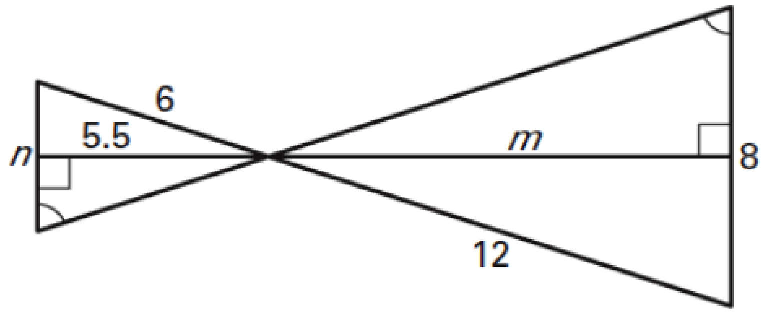
9. Find the perimeter of *MNOP*.

10. Find the ratio of the perimeter of *MNOP* to the perimeter of *WXYZ*.

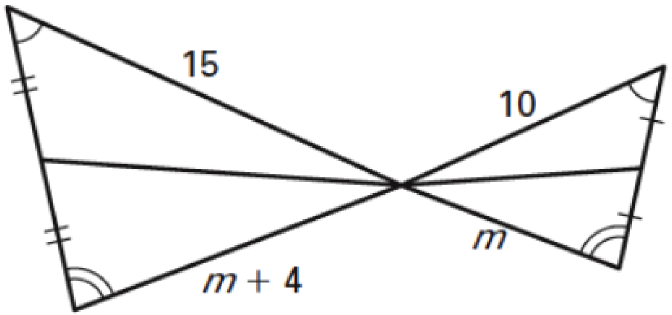


The two triangles are similar. Find the values of the variables.

11.



12.



In Exercises 13 and 14, use the following information.

Similar Triangles Triangles *RST* and *WXY* are similar. The side lengths of $\triangle RST$ are 10 inches, 14 inches, and 20 inches, and the length of an altitude is 6.5 inches. The shortest side of $\triangle WXY$ is 15 inches long.

13. Find the lengths of the other two sides of $\triangle WXY$.

14. Find the length of the corresponding altitude in $\triangle WXY$.

15. **Multiple Choice** The ratio of one side of $\triangle ABC$ to the corresponding side of a similar $\triangle DEF$ is 4 : 3. The perimeter of $\triangle DEF$ is 24 inches. What is the perimeter of $\triangle ABC$?

A. 18 inches **B.** 24 inches **C.** 32 inches

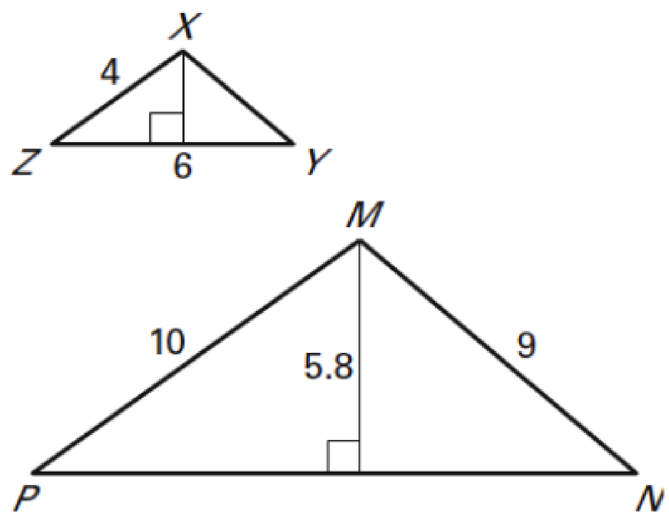
In the diagram, $\triangle XYZ \sim \triangle MNP$.

16. Find the scale factor of $\triangle XYZ$ to $\triangle MNP$.

17. Find the unknown side lengths of both triangles.

18. Find the length of the altitude shown in $\triangle XYZ$.

19. Find and compare the areas of both triangles.



In Exercises 20–22, use the following information.

Swimming Pool The community park has a rectangular swimming pool enclosed by a rectangular fence for sunbathing. The shape of the pool is similar to the shape of the fence. The pool is 30 feet wide. The fence is 50 feet wide and 100 feet long.

20. What is the scale factor of the pool to the fence?

21. What is the length of the pool?

22. Find the area reserved strictly for sunbathing.