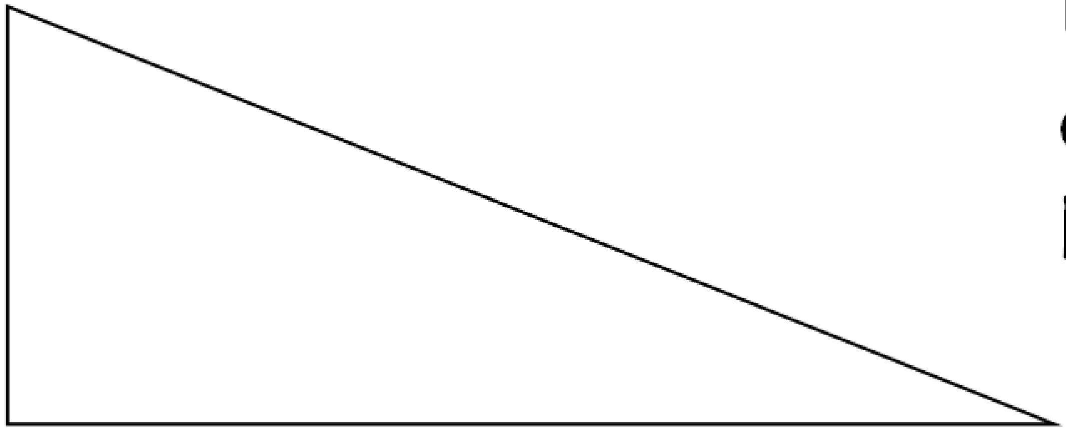
7.3 Use Similar Right Triangles

Goal • Use properties of the altitude of a right triangle.



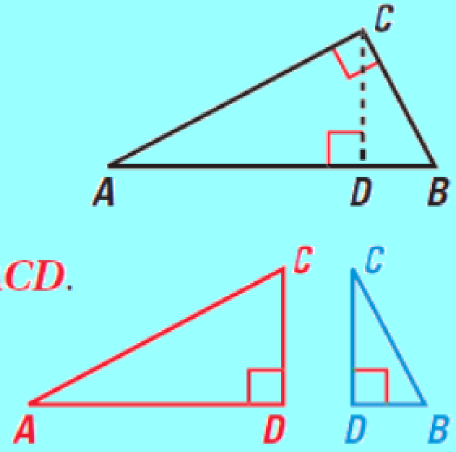
Draw an altitude to the hypotenuse.

THEOREM — *For Your Notebook*

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

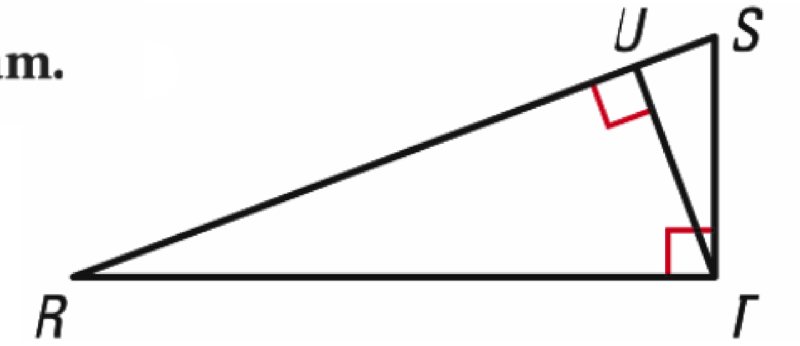
$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



Proof: below; Ex. 35, p. 456

EXAMPLE 1 Identify similar triangles

Identify the similar triangles in the diagram.



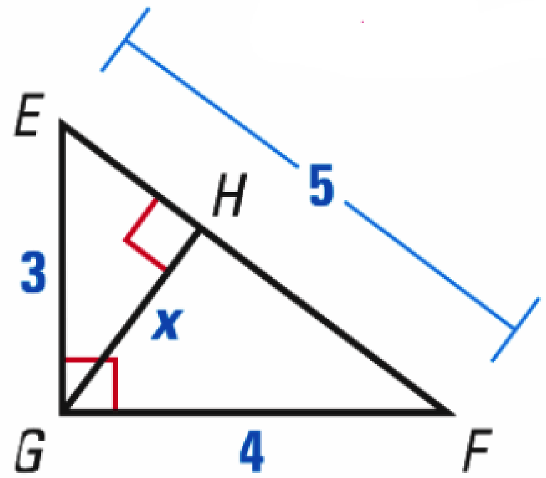
If you are struggling with seeing which angles are congruent and which triangles are similar.

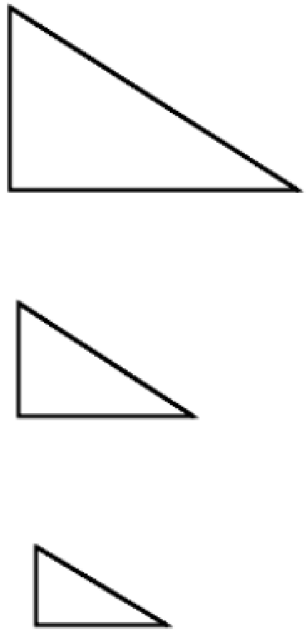
Draw the triangles out!

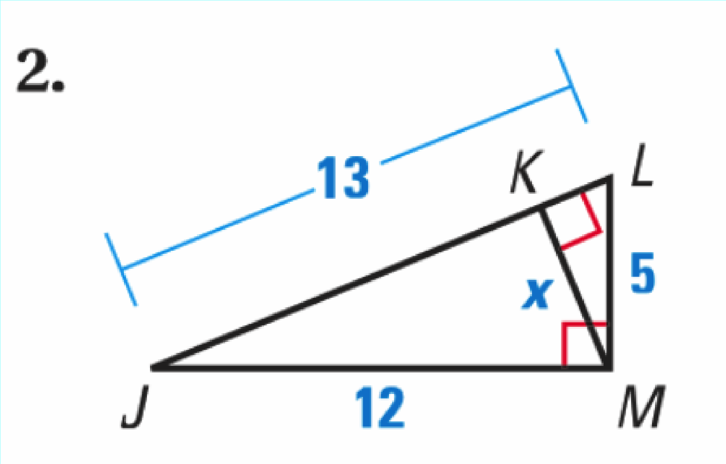
GUIDED PRACTICE for Examples 1 and 2

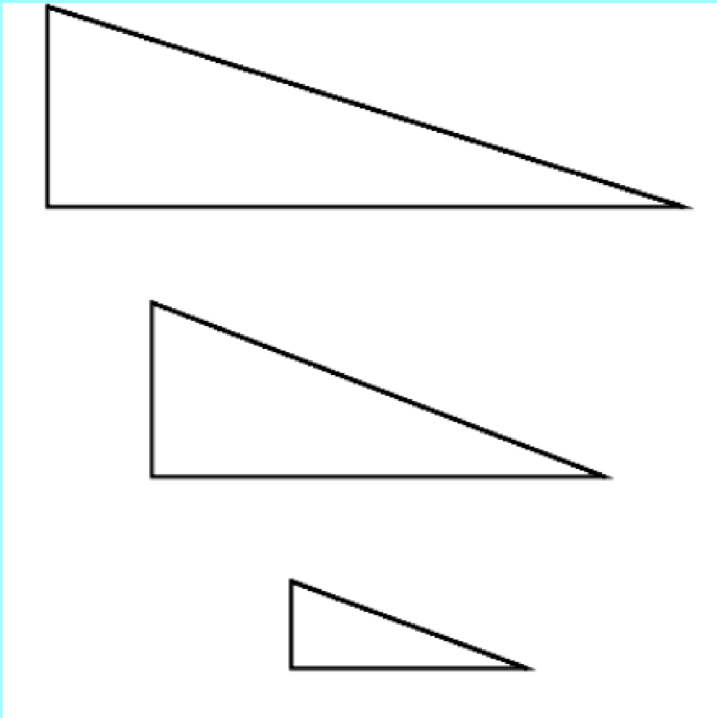
Identify the similar triangles. Then find the value of x.

1.



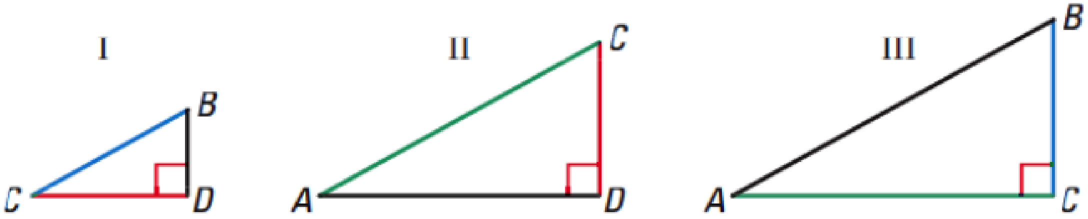



2.
13
K
L
x
5
J
12
M

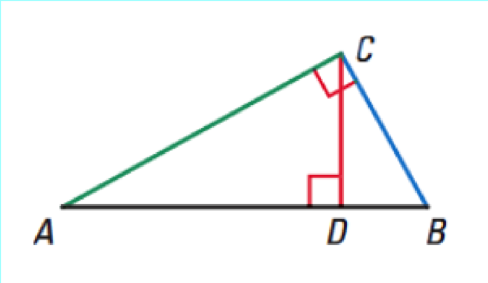


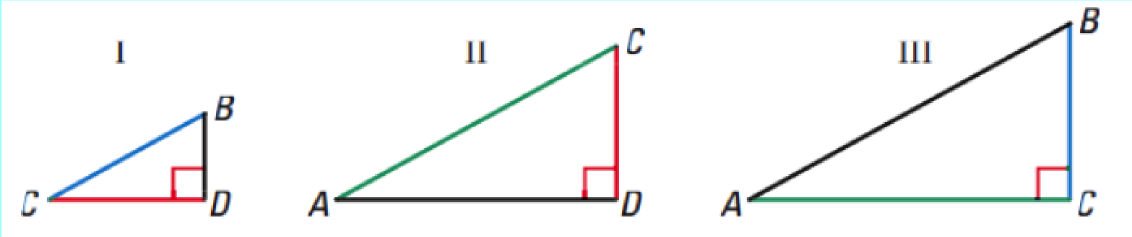
READ SYMBOLS Remember that an altitude is defined as a segment. So, $\overline{CD}$ refers to an altitude in $\triangle ABC$ and CD refers to its length.

GEOMETRIC MEANS In Lesson 6.1, you learned that the *geometric mean* of two numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. Consider right $\triangle ABC$. From Theorem 7.5, you know that altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.



Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that CD is the geometric mean of BD and AD. As you see below, CB and AC are also geometric means of segment lengths in the diagram.



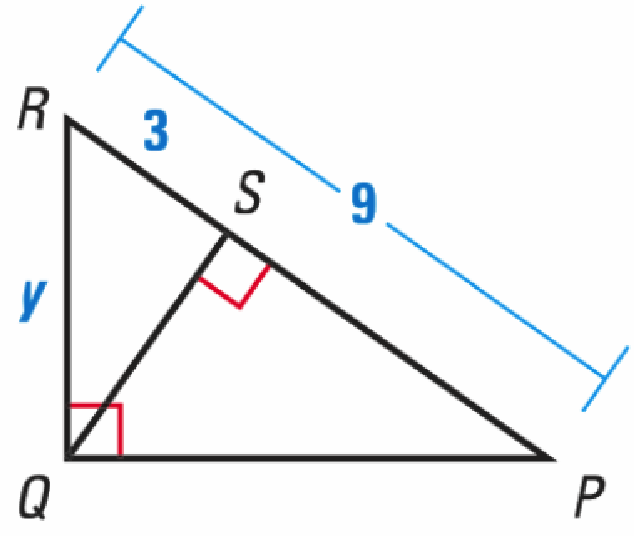


Proportions Involving Geometric Means in Right △ABC

length of shorter leg of I / length of shorter leg of II →	$\frac{BD}{CD} = \frac{CD}{AD}$	← length of longer leg of I / length of longer leg of II
length of hypotenuse of III / length of hypotenuse of I →	$\frac{AB}{CB} = \frac{CB}{DB}$	← length of shorter leg of III / length of shorter leg of I
length of hypotenuse of III / length of hypotenuse of II →	$\frac{AB}{AC} = \frac{AC}{AD}$	← length of longer leg of III / length of longer leg of II

EXAMPLE 3 Use a geometric mean

xy Find the value of *y*. Write your answer in simplest radical form.



WRITE PROOFS

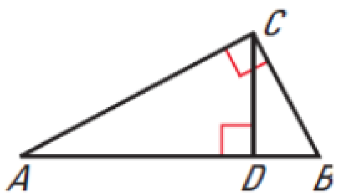
In Exercise 32 on page 455, you will use the geometric mean theorems to prove the Pythagorean Theorem.

THEOREMS *For Your Notebook*

THEOREM 7.6 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.



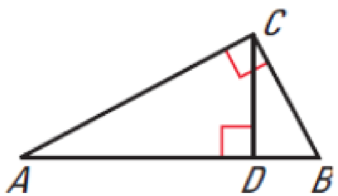
$$\frac{BD}{CD} = \frac{CD}{AD}$$

Proof: Ex. 36, p. 456

THEOREM 7.7 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

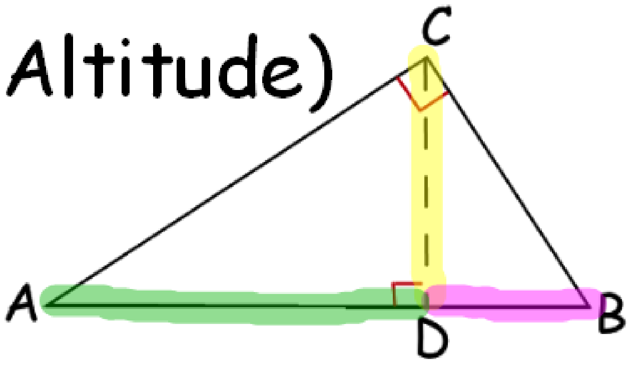


$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

Proof: Ex. 37, p. 456

Thm 7.6

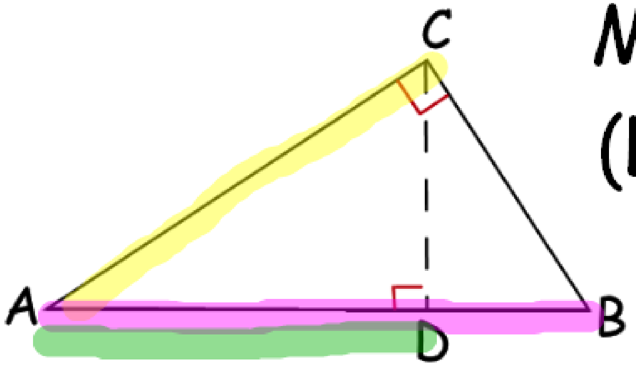
Geometric Mean (Altitude)

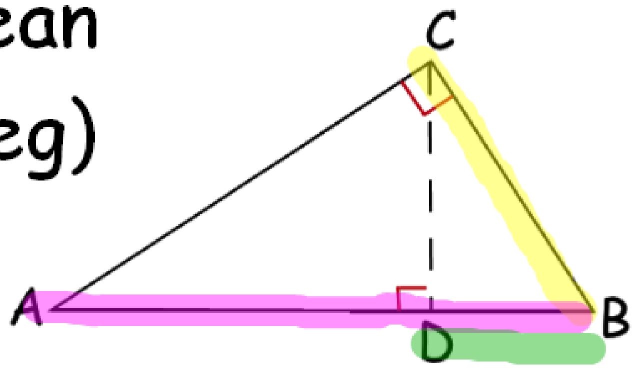


$$\frac{BD}{CD} = \frac{CD}{AD}$$

Thm 7.7

Geometric Mean (Leg)





$$\frac{AB}{AC} = \frac{AC}{AD}$$

$$\frac{AB}{CB} = \frac{CB}{DB}$$

Assignment Day 1:

7.3 ws

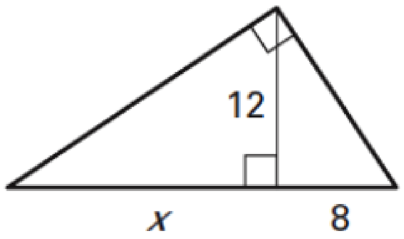
LESSON 7.3

Practice B

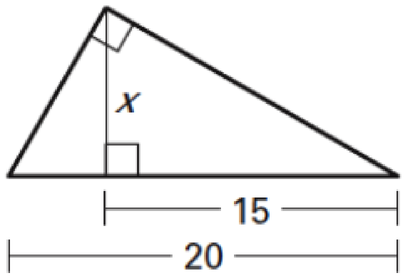
For use with pages 448–456

Complete and solve the proportion.

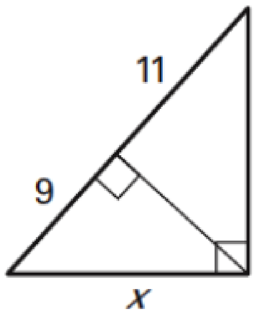
1. $\frac{x}{12} = \frac{?}{8}$



2. $\frac{15}{x} = \frac{x}{?}$

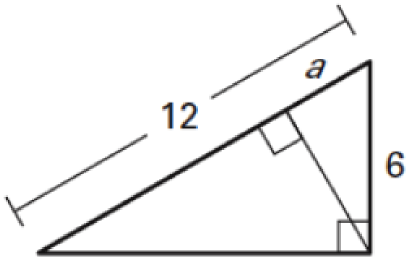


3. $\frac{9}{x} = \frac{x}{?}$

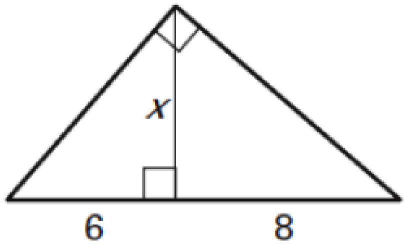


Find the value(s) of the variable(s).

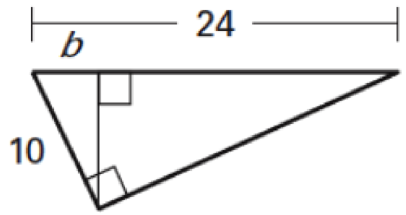
4.



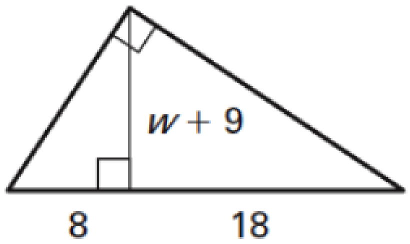
5.



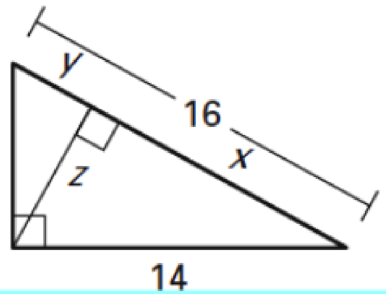
6.



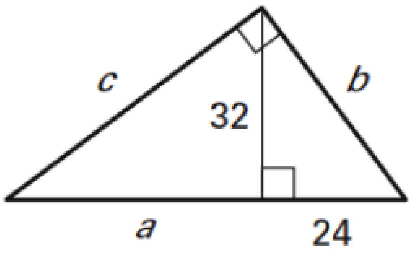
7.



8.

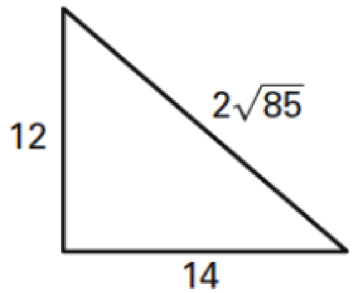


9.

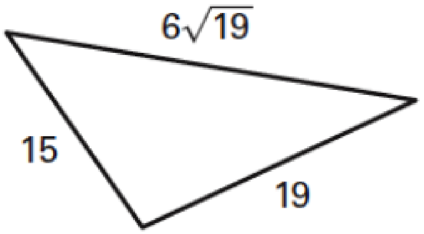


Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.

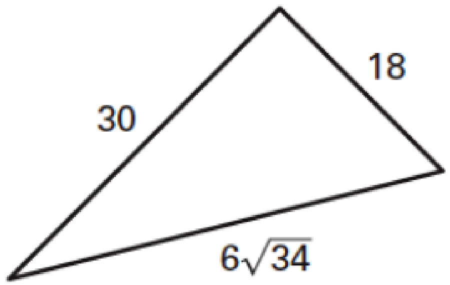
10.



11.

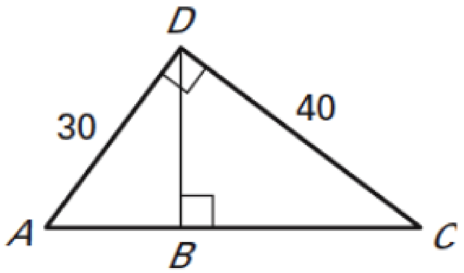


12.

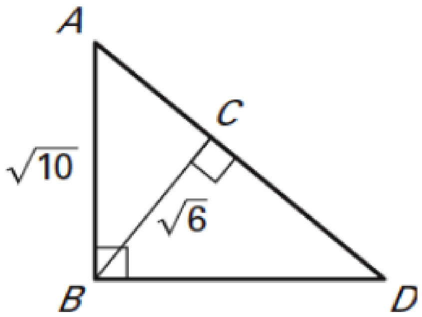


Use the Geometric Mean Theorems to find *AC* and *BD*.

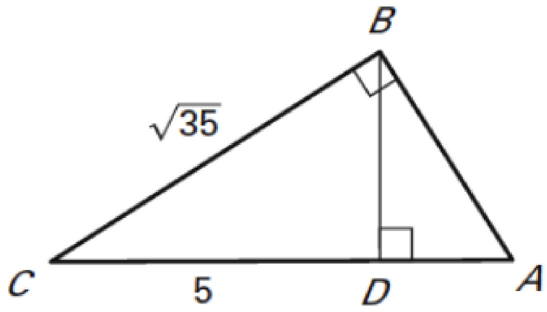
13.



14.



15.



LESSON 7.3

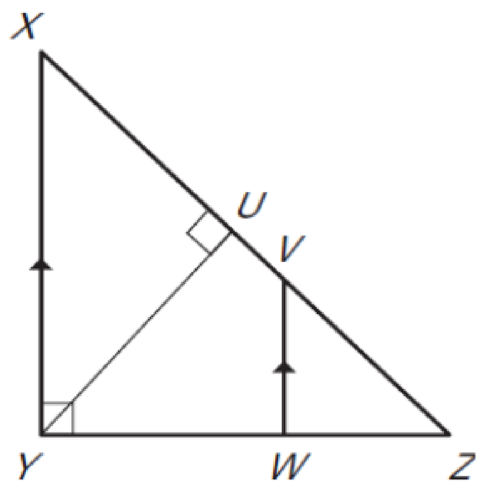
Practice B *continued*

For use with pages 448–456

16. Complete the proof.

GIVEN: $\triangle XYZ$ is a right triangle with $m\angle XYZ = 90°$.
$\overline{VW} \parallel \overline{XY}$, $\overline{YU}$ is an altitude of $\triangle XYZ$.

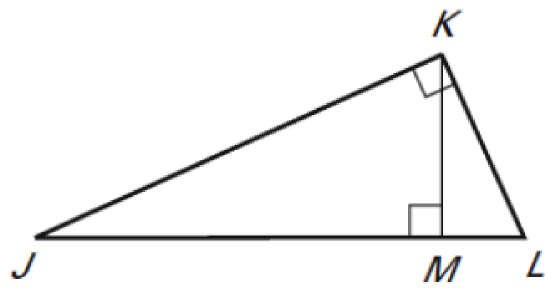
PROVE: $\triangle YUZ \sim \triangle VWZ$



Statements	Reasons
1. $\triangle XYZ$ is a right $\triangle$ with altitude $\overline{YU}$.	**1.** ___?___
2. $\triangle XYZ \sim \triangle YUZ$	**2.** ___?___
3. $\overline{VW} \parallel \overline{XY}$	**3.** ___?___
4. $\angle VWZ \cong \angle XYZ$	**4.** ___?___
5. $\angle Z \cong \angle Z$	**5.** ___?___
6. ___?___	**6.** AA Similarity Postulate
7. $\triangle YUZ \sim \triangle VWZ$	**7.** ___?___

In Exercises 17–19, use the diagram.

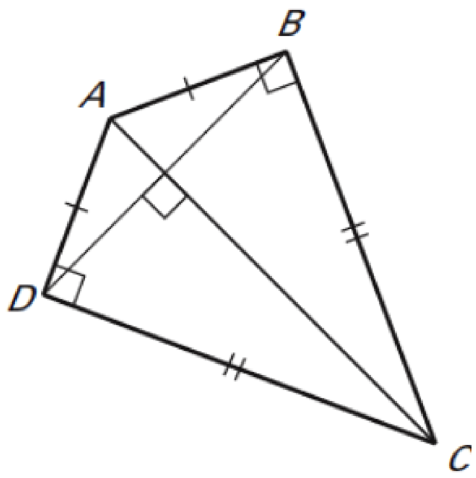
17. Sketch the three similar triangles in the diagram. Label the vertices.



18. Write similarity statements for the three triangles.

19. Which segment's length is the geometric mean of *LM* and *JM*?

20. Kite Design You are designing a diamond-shaped kite. You know that $AB = 38.4$ centimeters, $BC = 72$ centimeters, and $AC = 81.6$ centimeters. You want to use a straight crossbar $\overline{BD}$. About how long should it be?



Assignment Day 2:

p. 453
(1, 3-11, 13-18, 21, 23, 29, 30, 39-46)