7.7 Solve Right Triangles

Goal • Use inverse tangent, sine, and cosine ratios.

To **solve a right triangle** means to find the measures of all of its sides and angles. You can solve a right triangle if you know either of the following:

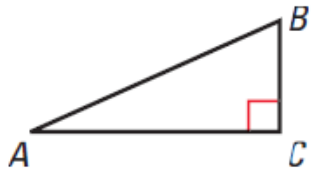
- Two side lengths
- One side length and the measure of one acute angle

In Lessons 7.5 and 7.6, you learned how to use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. Once you know the tangent, the sine, or the cosine of an acute angle, you can use a calculator to find the measure of the angle.

KEY CONCEPT *For Your Notebook*

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



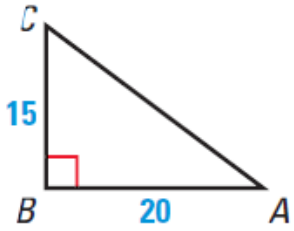
Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\tan^{-1} \frac{BC}{AC} = m\angle A$

READ VOCABULARY

The expression "$\tan^{-1} x$" is read as "the inverse

EXAMPLE 1 Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Solution

Because $\tan A = \frac{15}{20} = \frac{3}{4} = 0.75$, $\tan^{-1} 0.75 = m\angle A$. Use a calculator.

$\tan^{-1} 0.75 \approx 36.86989765 \cdots$

▶ So, the measure of $\angle A$ is approximately 36.9°.

EXAMPLE 2 Use an inverse sine and an inverse cosine

ANOTHER WAY

You can use the Table of Trigonometric Ratios on p. 925 to approximate $\sin^{-1} 0.87$ to the nearest degree. Find the number closest to 0.87 in the sine column and

Let $\angle A$ and $\angle B$ be acute angles in a right triangle. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

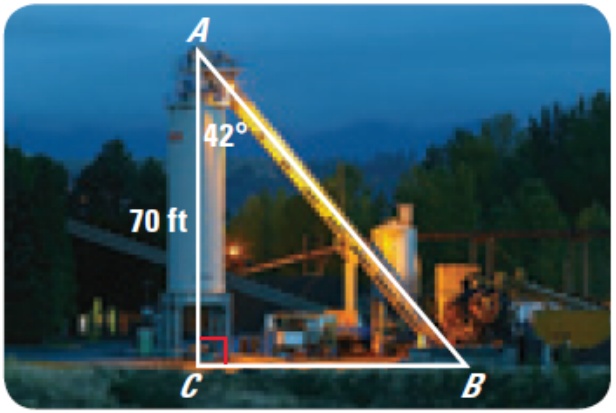
a. $\sin A = 0.87$ **b.** $\cos B = 0.15$

Solution

a. $m\angle A = \sin^{-1} 0.87 \approx 60.5°$ **b.** $m\angle B = \cos^{-1} 0.15 \approx 81.4°$

EXAMPLE 3 Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



Solution

STEP 1 **Find** $m\angle B$ by using the Triangle Sum Theorem.

STEP 2 **Approximate** BC by using a tangent ratio.

STEP 3 **Approximate** AB using a cosine ratio.

$$AB \approx \frac{70}{0.7431}$$ Use a calculator to find cos 42°.

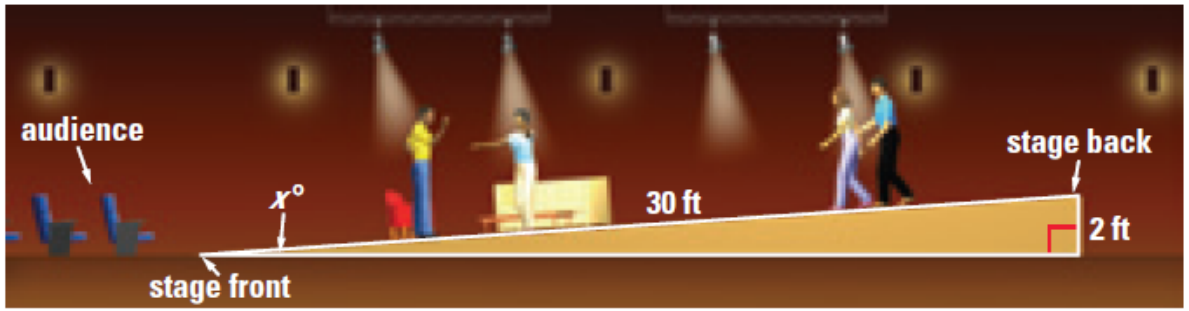
ANOTHER WAY

You could also find AB by using the Pythagorean Theorem, or a sine ratio.

EXAMPLE 4 Solve a real-world problem

READ VOCABULARY
A *raked stage* slants upward from front to back to give the audience a better view.

THEATER DESIGN Suppose your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?



Solution

LESSON 7.7

Practice

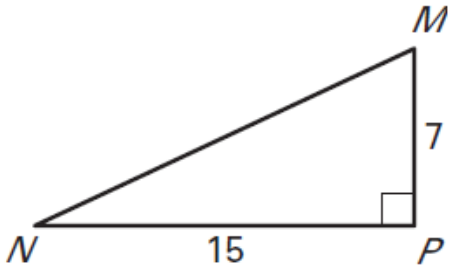
For use with pages 483–489

Use the diagram to find the indicated measurement. Round your answer to the nearest tenth.

1. MN

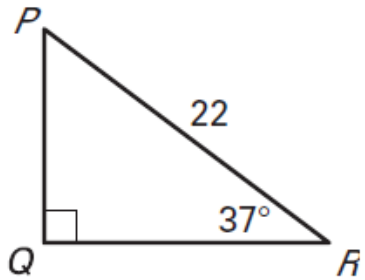
2. $m\angle M$

3. $m\angle N$



Solve the right triangle. Round decimal answers to the nearest tenth.

4.



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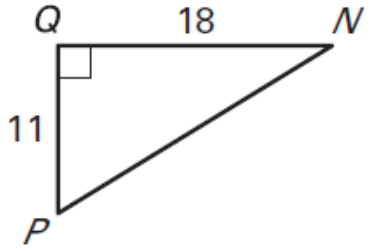

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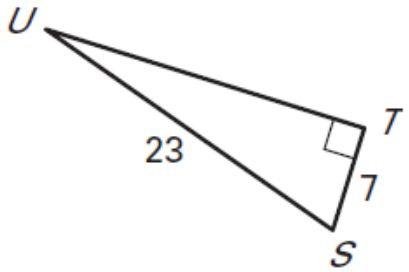


Assignment Day 1:

7.7 ws

LESSON 7.7

Practice B

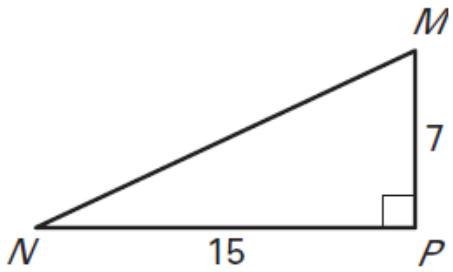
For use with pages 483–489

Use the diagram to find the indicated measurement. Round your answer to the nearest tenth.

1. MN

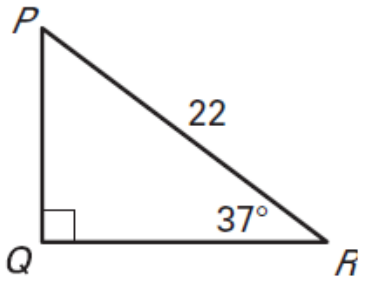
2. $m\angle M$

3. $m\angle N$



Solve the right triangle. Round decimal answers to the nearest tenth.

4.



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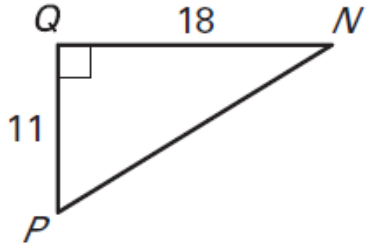

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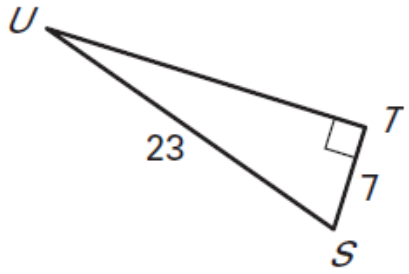

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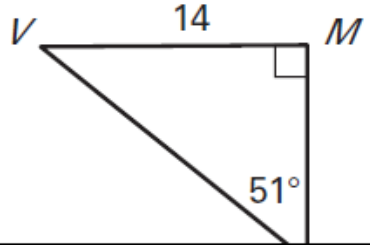

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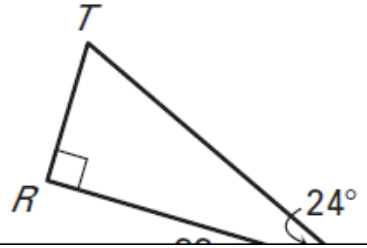

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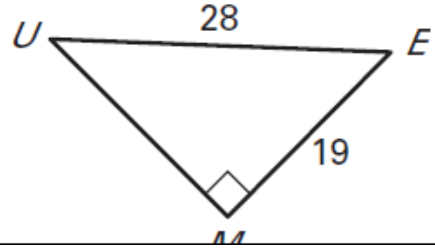

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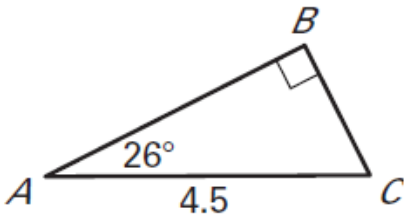

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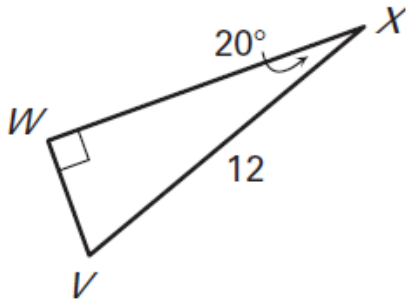

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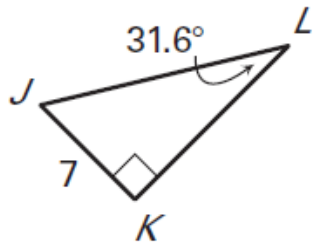


Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.

13. $\sin A = 0.36$

14. $\tan A = 0.8$

15. $\sin A = 0.27$

16. $\cos A = 0.35$

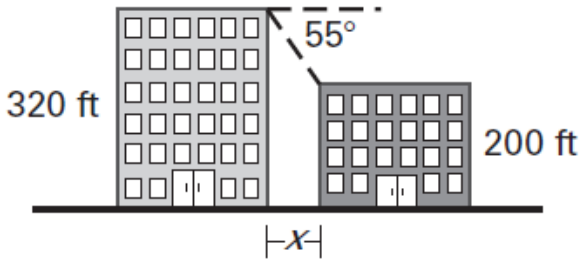
17. $\tan A = 0.42$

18. $\cos A = 0.11$

19. $\sin A = 0.94$

20. $\cos A = 0.77$

21. Office Buildings The angle of depression from the top of a 320 foot office building to the top of a 200 foot office building is 55°. How far apart are the buildings?

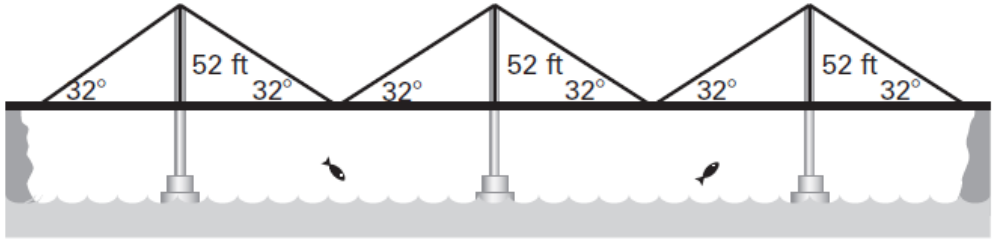


LESSON 7.7

Practice B *continued*

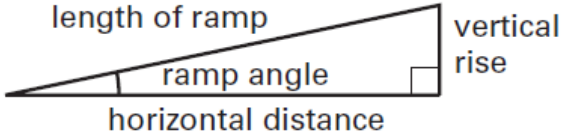
For use with pages 483–489

22. Suspension Bridge Use the diagram to find the distance across the suspension bridge.



In Exercises 23 and 24, use the following information.

Ramps The Uniform Federal Accessibility Standards specify that the ramp angle used for a wheelchair ramp must be less than or equal to 4.78°.

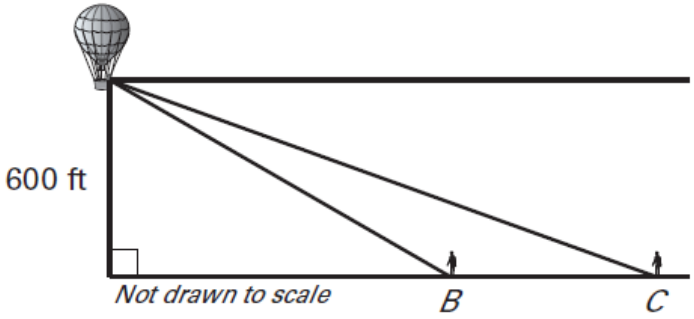


23. The length of one ramp is 16 feet. The vertical rise is 14 inches. Estimate the ramp's horizontal distance and its ramp angle. Does this ramp meet the Uniform Federal Accessibility Standards?

24. You want to build a ramp with a vertical rise of 6 inches. You want to minimize the horizontal distance taken up by the ramp. Draw a sketch showing the approximate dimensions of your ramp.

In Exercises 25–27, use the following information.

Hot Air Balloon You are in a hot air balloon that is 600 feet above the ground where you can see two people.



25. If the angle of depression from your line of sight to the person at B is 30°, how far is the person from the point on the ground below the hot air balloon?

26. If the angle of depression from your line of sight to the person at C is 20°, how far is the person from the point on the ground below the hot air balloon?

27. How far apart are the two people?

Answer Key

Lesson 7.7

Practice Level B

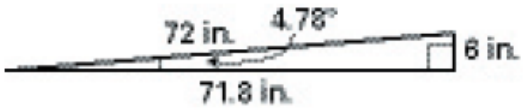
1. 16.6 **2.** 65 **3.** 25 **4.** $m\angle P = 53°$, $PQ \approx 13.2$, $QR \approx 17.6$ **5.** $m\angle P \approx 58.6°$, $m\angle N \approx 31.4°$, $PN \approx 21.1$ **6.** $TU \approx 21.9$, $m\angle S = 72.3°$, $m\angle U = 17.7°$ **7.** $m\angle V = 39°$, $DM \approx 11.3$, $DV \approx 18.0$ **8.** $m\angle T = 66°$, $TR \approx 14.7$, $AT \approx 36.1$ **9.** $UM \approx 20.6$, $m\angle U \approx 42.7°$, $m\angle E \approx 47.3°$ **10.** $m\angle C = 64°$, $AB \approx 4.0$, $BC \approx 2.0$ **11.** $m\angle V = 70°$, $VW \approx 4.1$, $WX \approx 11.3$ **12.** $m\angle J = 58.4°$, $JL \approx 13.4$, $LK \approx 11.4$ **13.** 21.1° **14.** 38.7°

15. 15.7° **16.** 69.5° **17.** 22.8° **18.** 83.7°

19. 70.1° **20.** 39.6° **21.** about 84.02 ft

22. about 499.30 ft **23.** about 191.5 in. or about 15 ft 11.5 in.; about 4.2°; Yes, the angle is less than 4.78°.

24.



25. about 1039.2 ft **26.** about 1648.5 ft

Assignment Day 2:

p. 485 (3-28, 43-48 all)