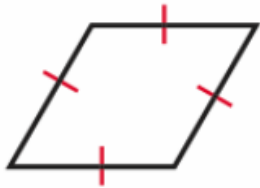


8.4 Properties of Rhombuses, Rectangles, and Squares

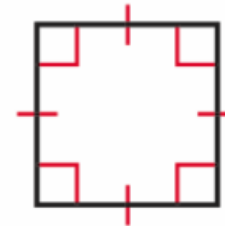
In this lesson, you will learn about three special types of parallelograms: *rhombuses*, *rectangles*, and *squares*.



A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

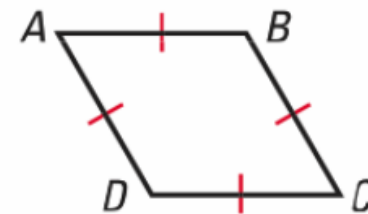
You can use the corollaries below to prove that a quadrilateral is a rhombus, rectangle, or square, without first proving that the quadrilateral is a parallelogram.

COROLLARIES*For Your Notebook***RHOMBUS COROLLARY**

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof: Ex. 57, p. 539

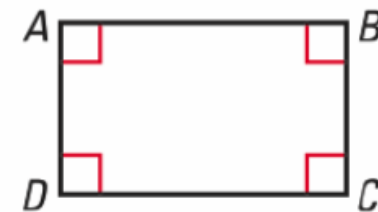


RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof: Ex. 58, p. 539

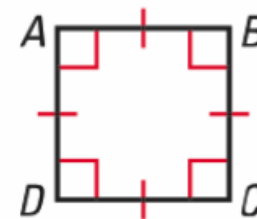


SQUARE COROLLARY

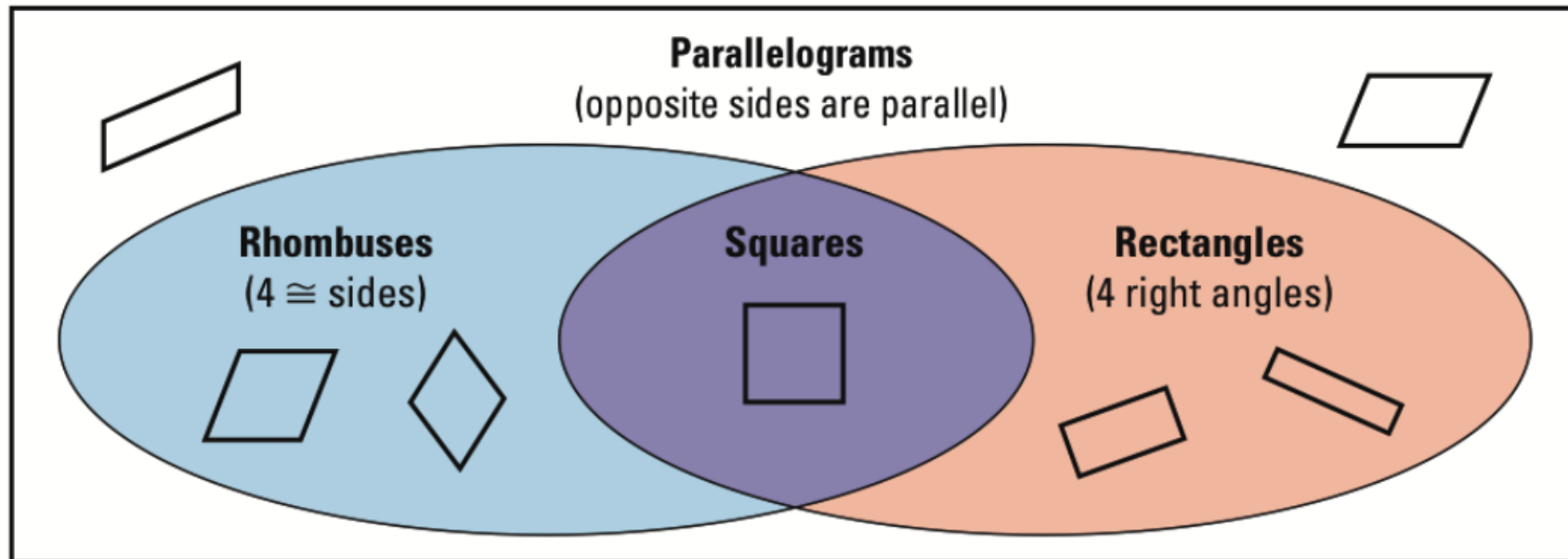
A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof: Ex. 59, p. 539



The *Venn diagram* below illustrates some important relationships among parallelograms, rhombuses, rectangles, and squares. For example, you can see that a square is a rhombus because it is a parallelogram with four congruent sides. Because it has four right angles, a square is also a rectangle.



EXAMPLE 1 Use properties of special quadrilaterals

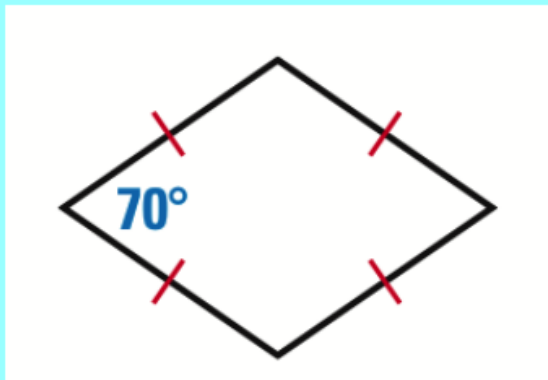
For any rhombus $QRST$, decide whether the statement is *always* or *sometimes* true. Draw a sketch and explain your reasoning.

a. $\angle Q \cong \angle S$

b. $\angle Q \cong \angle R$

EXAMPLE 2 Classify special quadrilaterals

Classify the special quadrilateral. Explain your reasoning.



DIAGONALS The theorems below describe some properties of the diagonals of rhombuses and rectangles.

THEOREMS

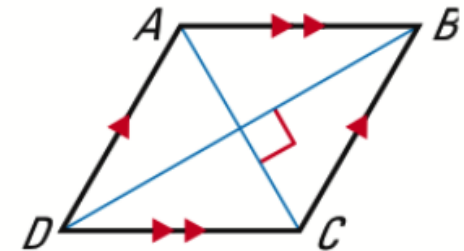
For Your Notebook

THEOREM 8.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof: p. 536; Ex. 56, p. 539

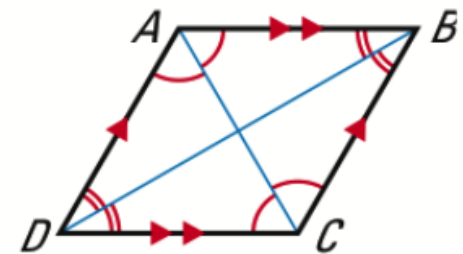


THEOREM 8.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if \overline{AC} bisects $\angle BCD$ and $\angle BAD$ and \overline{BD} bisects $\angle ABC$ and $\angle ADC$.

Proof: Exs. 60–61, p. 539

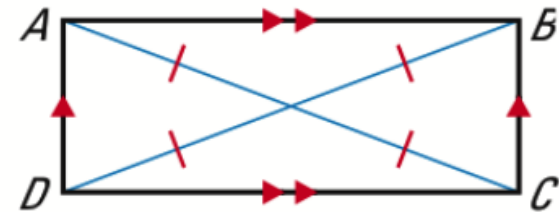


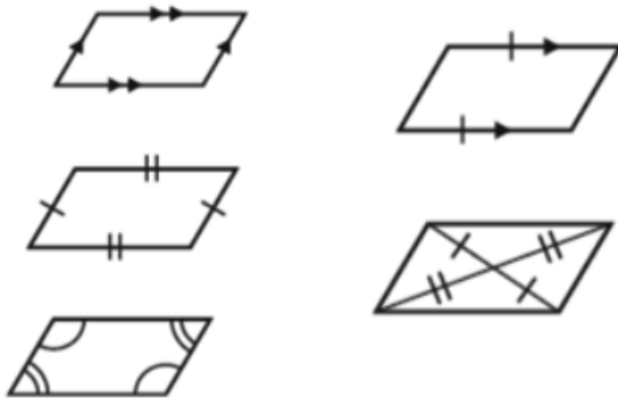
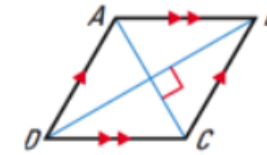
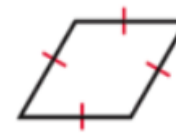
THEOREM 8.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

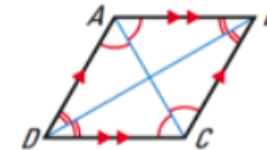
$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof: Exs. 63–64, p. 540

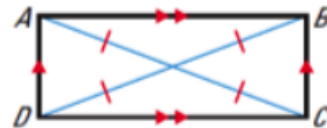
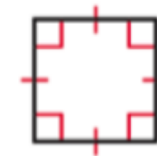


Parallelogram Characteristics:Rhombus Characteristics:

A rhombus is a parallelogram with four congruent sides.

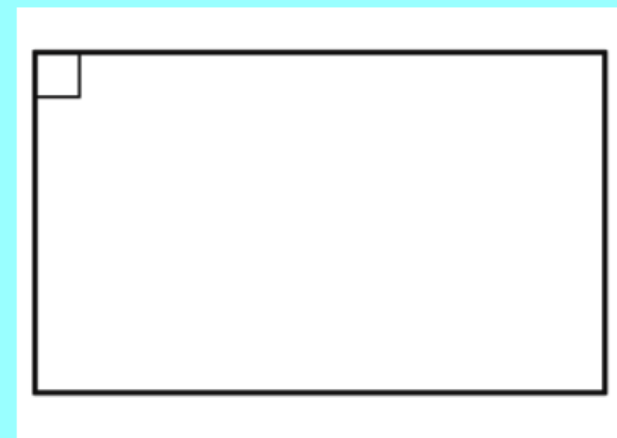
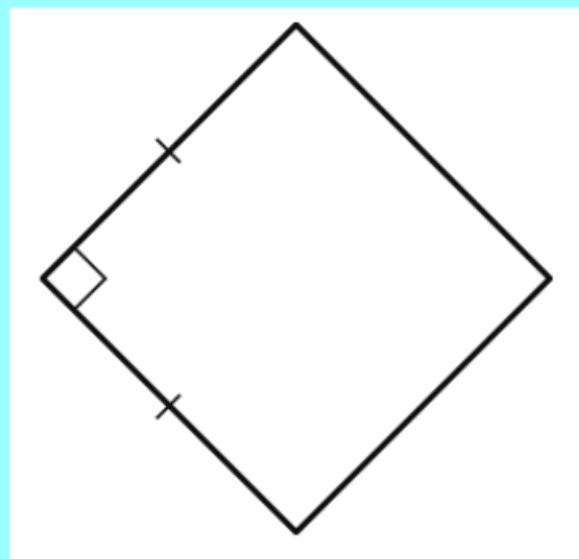
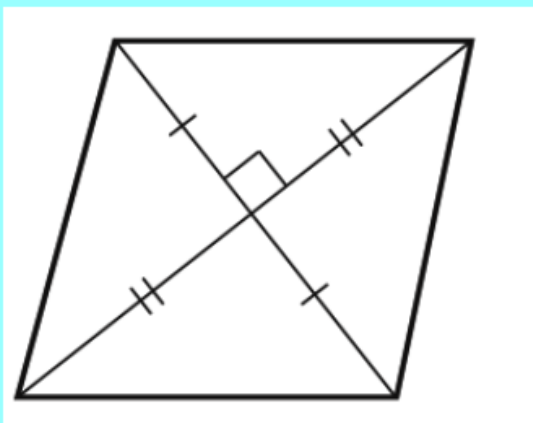
Rectangle Characteristics:

A rectangle is a parallelogram with four right angles.

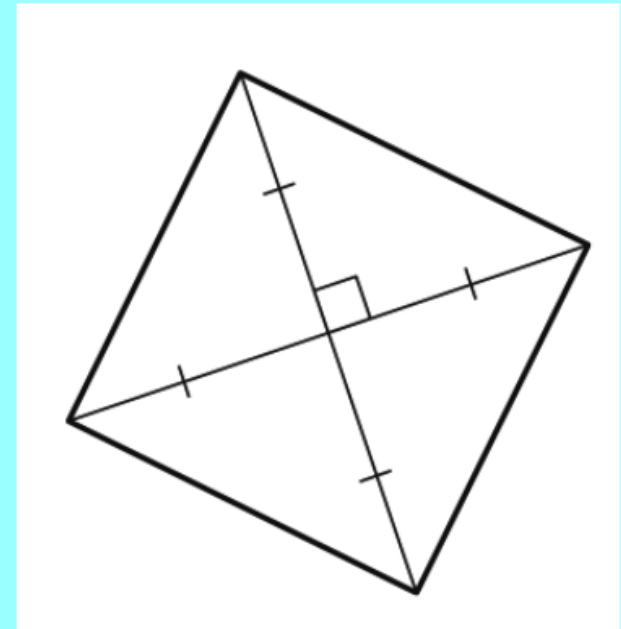
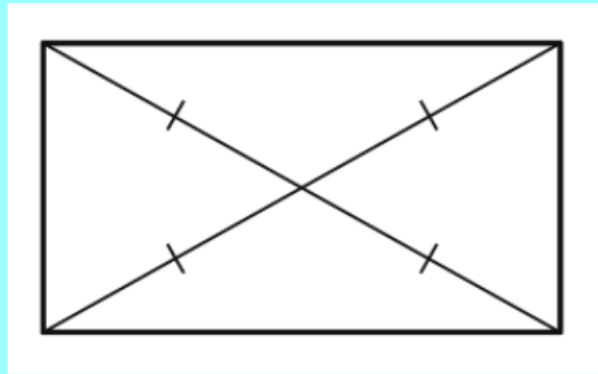
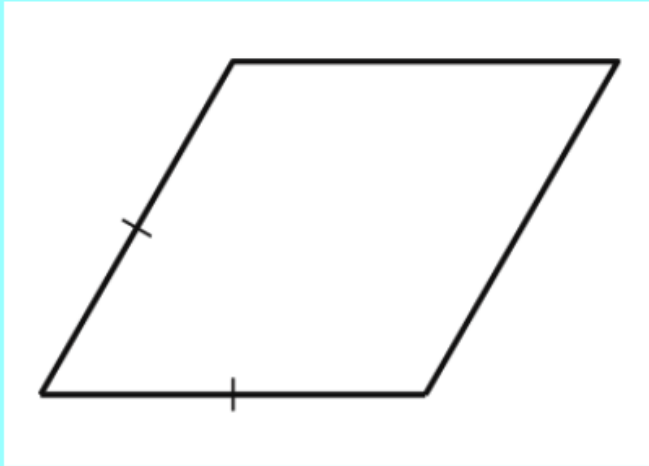
Square Characteristics:

A square is a parallelogram with four congruent sides and four right angles.

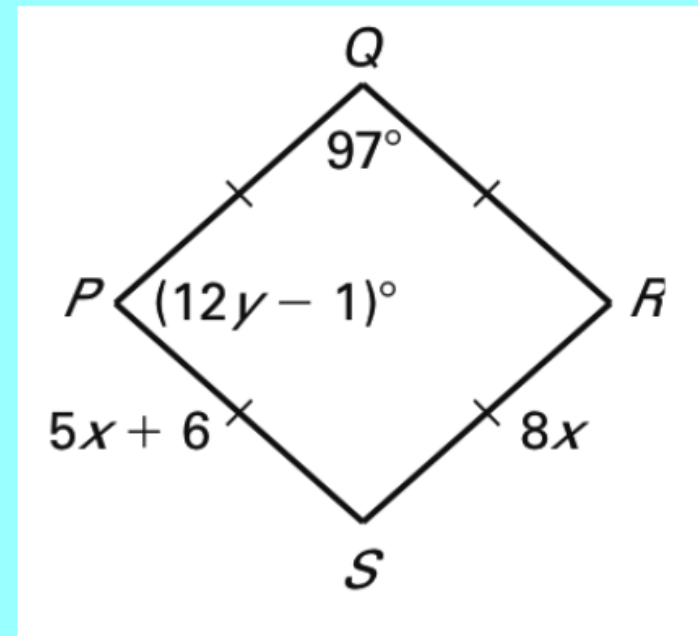
Classify the parallelogram. *Explain* your reasoning.



Classify the parallelogram. *Explain* your reasoning.



Classify the special quadrilateral. *Explain* your reasoning. Then find the values of x and y .



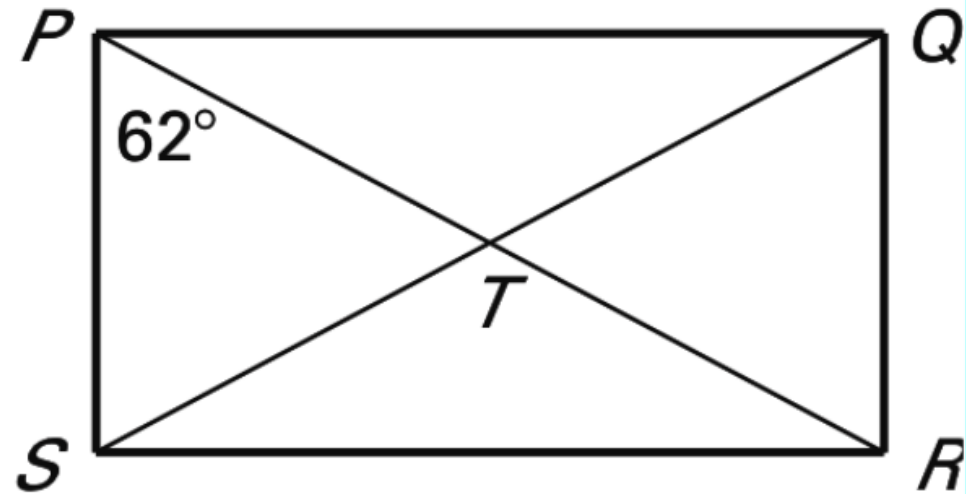
The diagonals of rectangle $PQRS$ intersect at T .
Given that $m\angle RPS = 62^\circ$ and $QS = 18$, find the indicated measure.

17. $m\angle QPR$

18. $m\angle PTQ$

19. ST

20. PR



Assignment

8.4 WS

LESSON
8.4**Practice B***For use with pages 533–540*

For any rhombus $ABCD$, decide whether the statement is *always* or *sometimes* true. Draw a diagram and *explain* your reasoning.

1. $\angle ABC \cong \angle CDA$

2. $\overline{CA} \cong \overline{DB}$

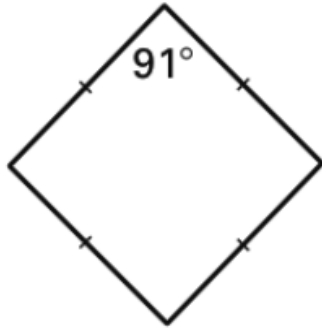
For any rectangle $FGHJ$, decide whether the statement is *always* or *sometimes* true. Draw a diagram and *explain* your reasoning.

3. $\angle F \cong \angle H$

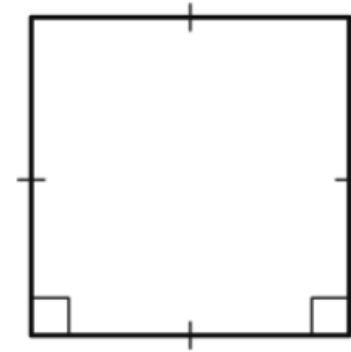
4. $\overline{GH} \cong \overline{HJ}$

Classify the quadrilateral. *Explain* your reasoning.

5.



6.



Name each quadrilateral—*parallelogram, rectangle, rhombus, and square*—for which the statement is true.

7. It is equilateral.

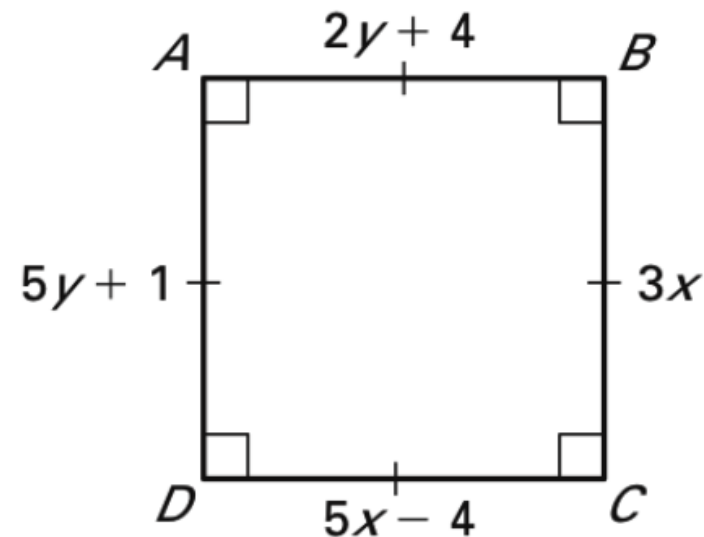
8. The diagonals are congruent.

9. It can contain obtuse angles.

10. It contains no acute angles.

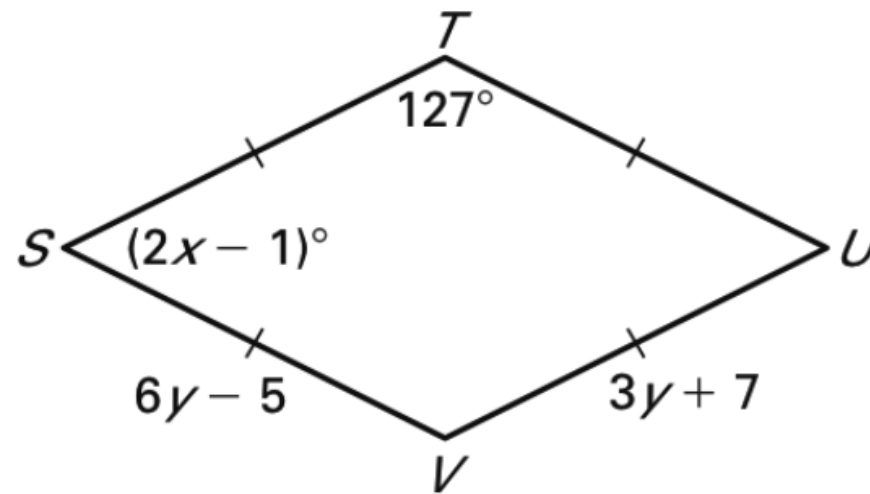
Classify the special quadrilateral. *Explain* your reasoning. Then find the values of x and y .

11.



Classify the special quadrilateral. *Explain* your reasoning. Then find the values of x and y .

12.



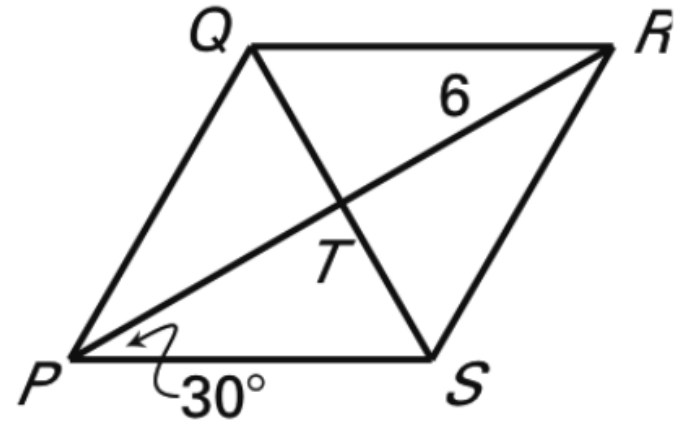
The diagonals of rhombus $PQRS$ intersect at T . Given that $m\angle RPS = 30^\circ$ and $RT = 6$, find the indicated measure.

13. $m\angle QPR$

14. $m\angle QTP$

15. RP

16. QT



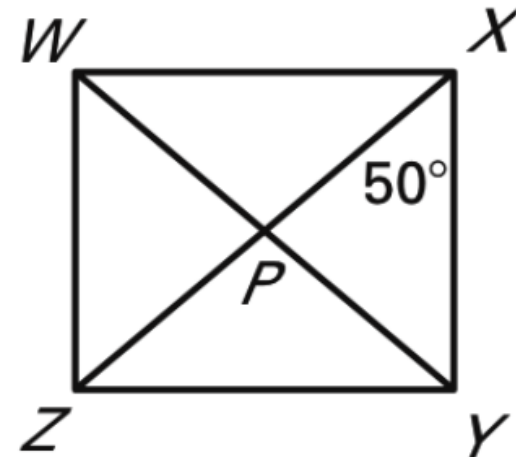
The diagonals of rectangle $WXYZ$ intersect at P . Given that $m\angle YXZ = 50^\circ$ and $XZ = 12$, find the indicated measure.

17. $m\angle WXZ$

18. $m\angle WPX$

19. PY

20. WX



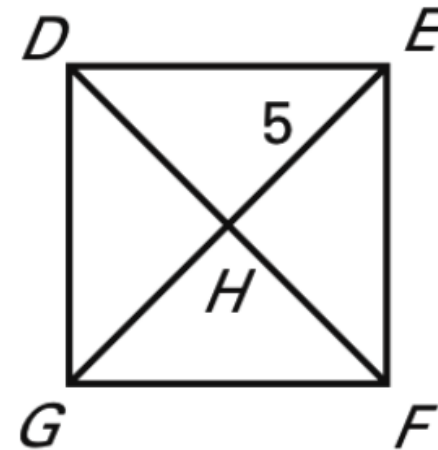
The diagonals of square $DEFG$ intersect at H .
Given that $EH = 5$, find the indicated measure.

21. $m\angle GHF$

22. $m\angle DGH$

23. HF

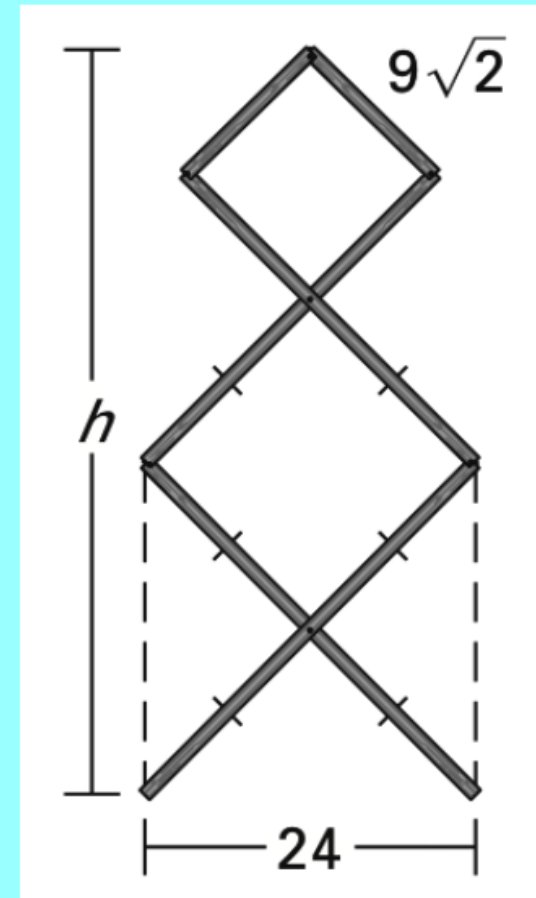
24. DE



25. Windows In preparation for a storm, a window is protected by nailing boards along its diagonals. The lengths of the boards are the same. Can you conclude that the window is square? *Explain.*

26. Clothing The side view of a wooden clothes dryer is shown at the right. Measurements shown are in inches.

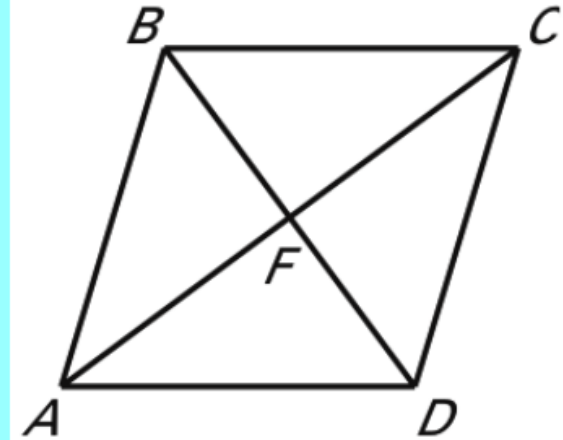
- The uppermost quadrilateral is a square. Classify the quadrilateral below the square. *Explain* your reasoning.
- Find the height h of the clothes dryer.



27. Proof The diagonals of rhombus $ABCD$ form several triangles. Using a two-column proof, prove that $\triangle BFA \cong \triangle DFC$.

GIVEN: $ABCD$ is a rhombus.

PROVE: $\triangle BFA \cong \triangle DFC$



Assignment:

p. 537 (3-17, 19-24, 32-49, 68-70 all)