9.2 Use Properties of Matrices

A **matrix** is a rectangular arrangement of numbers in rows and columns. (The plural of matrix is *matrices*.) Each number in a matrix is called an **element**.

column

row $\begin{bmatrix} 5 & 4 & 4 & 9 \\ -3 & 5 & 2 & 6 \\ 3 & -7 & 8 & 7 \end{bmatrix}$ ← **The element in the second row and third column is 2.**

The **dimensions** of a matrix are the numbers of rows and columns. The matrix above has three rows and four columns, so the dimensions of the matrix are 3×4 (read "3 by 4").

You can represent a figure in the coordinate plane using a matrix with two rows. The first row has the x-coordinate(s) of the vertices. The second row has the corresponding y-coordinate(s). Each column represents a vertex, so the number of columns depends on the number of vertices of the figure.

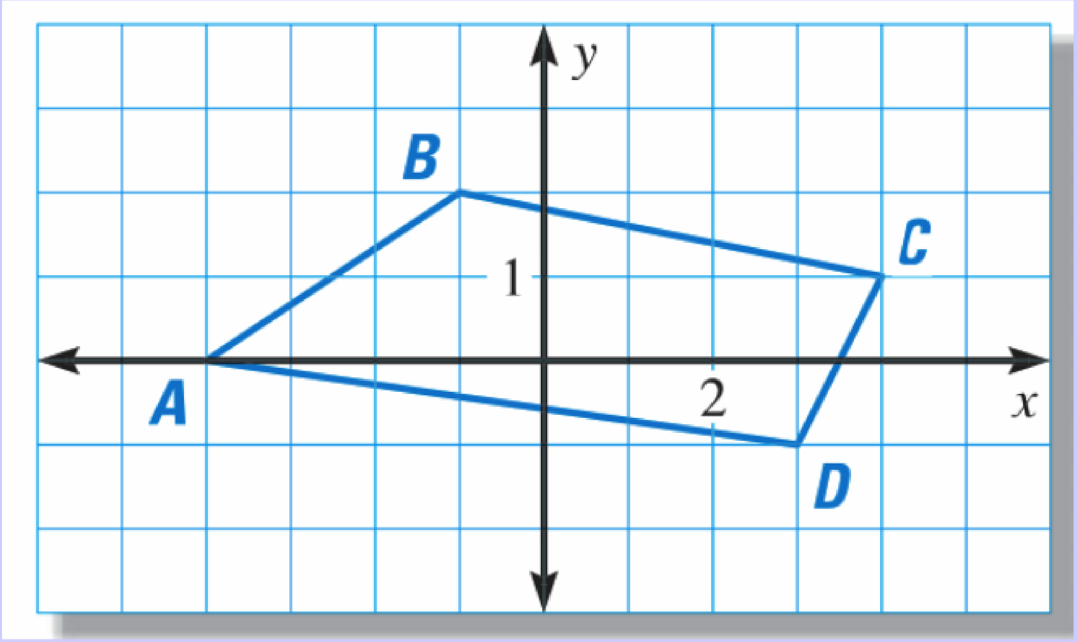
Make a matrix showing the following points:

A(2,-4), B(-6,1), and C(0,-3)

EXAMPLE 1 Represent figures using matrices

Write a matrix to represent the point or polygon.

a. Point *A*



b. Quadrilateral *ABCD*

To ADD or SUBTRACT matrices:

1) The matrices must have the same dimensions!

2) Add or subtract corresponding elements.

EXAMPLE 2 Add and subtract matrices

a. $\begin{bmatrix} 5 & -3 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$

b. $\begin{bmatrix} 6 & 8 & 5 \\ 4 & 9 & -1 \end{bmatrix} - \begin{bmatrix} 1 & -7 & 0 \\ 4 & -2 & 3 \end{bmatrix}$

EXAMPLE 3 Represent a translation using matrices

The matrix $\begin{bmatrix} 1 & 5 & 3 \\ 1 & 0 & -1 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 1 unit left and 3 units up. Then graph $\triangle ABC$ and its image.

$$\begin{bmatrix} -3 & 7 \end{bmatrix} + \begin{bmatrix} 2 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix}$$

MULTIPLYING MATRICES The product of two matrices A and B is defined only when the number of columns in A is equal to the number of rows in B. If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

$$A \cdot B = AB$$

$$(m \text{ by } n) \cdot (n \text{ by } p) = (m \text{ by } p)$$

equal

dimensions of ***AB***

You will use matrix multiplication in later lessons to represent transformations.

Multiplication of matrices is a little trickier!!

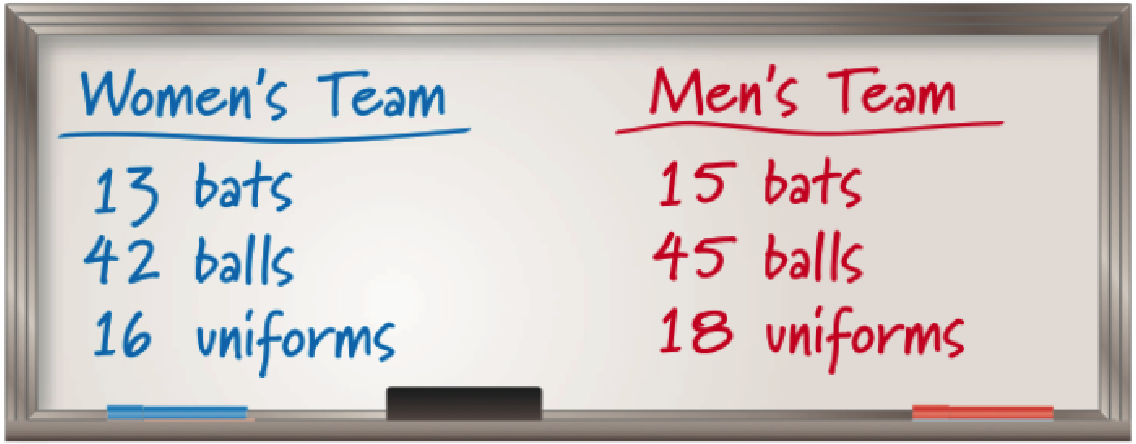
**If you have not taken notes already...
you will want to for this problem!!!

EXAMPLE 4 Multiply matrices

Multiply $\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix}$.

EXAMPLE 5 Solve a real-world problem

SOFTBALL Two softball teams submit equipment lists for the season. A bat costs $20, a ball costs $5, and a uniform costs $40. Use matrix multiplication to find the total cost of equipment for each team.



Matrices are set up on the next page!

EQUIPMENT · COST = TOTAL COST

$$\begin{array}{c} \\ \text{Women} \\ \text{Men} \end{array} \overset{\begin{array}{ccc}\text{Bats} & \text{Balls} & \text{Uniforms}\end{array}}{\begin{bmatrix} 13 & 42 & 16 \\ 15 & 45 & 18 \end{bmatrix}} \cdot \begin{array}{c} \text{Bats} \\ \text{Balls} \\ \text{Uniforms} \end{array} \overset{\text{Dollars}}{\begin{bmatrix} 20 \\ 5 \\ 40 \end{bmatrix}} = \begin{array}{c} \text{Women} \\ \text{Men} \end{array} \overset{\text{Dollars}}{\begin{bmatrix} ? \\ ? \end{bmatrix}}$$

$$\begin{bmatrix} 5 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 5 & 1 \end{bmatrix}$$

Assignment:

9.2A ws

1. $$\begin{bmatrix} 3 & 6 \\ -1 & -3 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 6 & 0 \\ 2 & 3 \end{bmatrix}$$

2. $$\begin{bmatrix} -5 & 2 & -2 \\ 4 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -5 & -6 \\ 1 & 3 & -3 \end{bmatrix}$$

3. $-5\begin{bmatrix} 5 & 6 & -4 \\ 4 & -2 & -1 \end{bmatrix}$

4. $-2u\begin{bmatrix} 7u & 3w^2 & 5u & 5 \end{bmatrix}$

5. $$\begin{bmatrix} -4n & n+m \\ -2n & -4n \end{bmatrix} + \begin{bmatrix} 4 & -5 \\ 3m & 0 \end{bmatrix}$$

6. $$\begin{bmatrix} -6r+t \\ -r \\ 6s \end{bmatrix} + \begin{bmatrix} 6r \\ -4t \\ -3r+2 \end{bmatrix}$$

7. $5\begin{bmatrix}6 & 1 & 2 & -6\end{bmatrix} - \begin{bmatrix}1 & 6 & -6 & 6\end{bmatrix}$

8. $\begin{bmatrix} 5 & 3 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} -6 & 0 \\ 1 & -4 \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ -2 & -6 \end{bmatrix}$

9. $\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -5 & 4 \end{bmatrix}$

11. $\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix}$

12. $\begin{bmatrix} -5 \\ 6 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \end{bmatrix}$

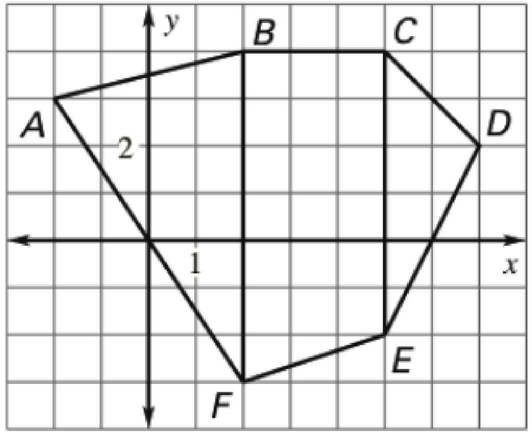
13. $\begin{bmatrix} 2 & -5v \end{bmatrix} \cdot \begin{bmatrix} -5u & -v \\ 0 & 6 \end{bmatrix}$

14. $\begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

Assignment Day 2:

9.2B ws

Use the diagram to write a matrix to represent the polygon.



1. $\triangle CDE$

2. $\triangle ABF$

3. Quadrilateral *BCEF*

4. Hexagon *ABCDEF*

Add or subtract.

5. $\begin{bmatrix} 6 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 9 \end{bmatrix}$

6. $\begin{bmatrix} -8 & 4 \\ 4 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 6 & -1 \end{bmatrix}$

7. $\begin{bmatrix} 5 & -2 \\ 2 & 4 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 6 & -4 \\ 6 & -1 \end{bmatrix}$

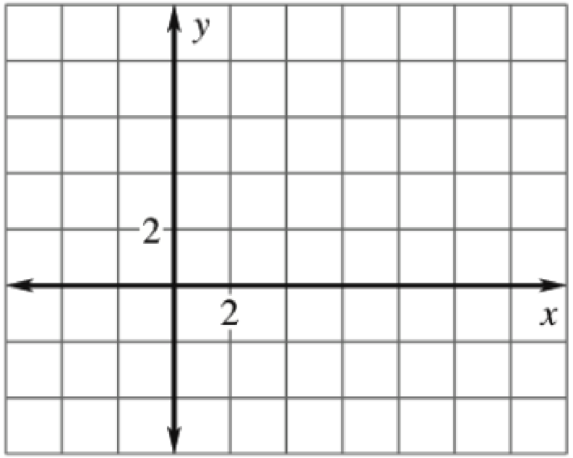
8. $\begin{bmatrix} -0.3 & 1.8 \end{bmatrix} - \begin{bmatrix} 0.6 & 2.7 \end{bmatrix}$

9. $\begin{bmatrix} -1 & -9 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 9 \\ -6 & -7 \end{bmatrix}$

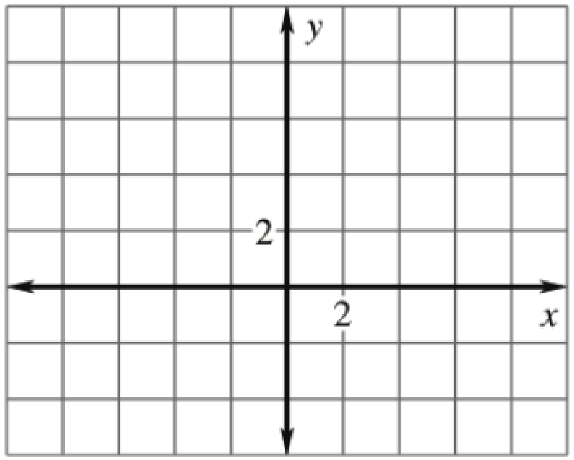
10. $\begin{bmatrix} 1.4 & 1.3 \\ -5 & -6.5 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -1.4 & -3 \\ 3.9 & 4 \\ 1.3 & 3.9 \end{bmatrix}$

Find the image matrix that represents the translation of the polygon. Then graph the polygon and its image.

11. $\begin{array}{ccc} A & B & C \end{array}$ $\begin{bmatrix} -1 & 5 & 3 \\ 2 & 2 & 6 \end{bmatrix}$; 5 units right and 3 units down



12. $\begin{matrix} M & N & O & P \end{matrix}$ $\begin{bmatrix} 3 & 7 & 5 & 1 \\ 1 & 2 & 6 & 5 \end{bmatrix}$; 6 units left and 2 units up



Multiply.

13. $\begin{bmatrix} 4 & -3 \end{bmatrix} \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

14. $\begin{bmatrix} -0.8 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ -1.6 \end{bmatrix}$

15. $\begin{bmatrix} -2 & 3 \\ 5 & -4 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 7 & 5 \end{bmatrix}$

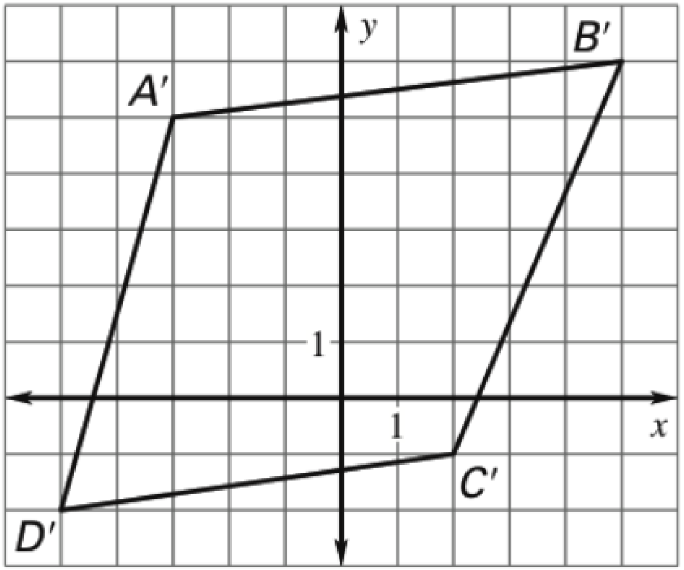
16. $\begin{bmatrix} 0.9 & 5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -4 & -3 \end{bmatrix}$

17. $\begin{bmatrix} -3 & 2 & 6 \end{bmatrix} \begin{bmatrix} -5 \\ 0 \\ -3 \end{bmatrix}$

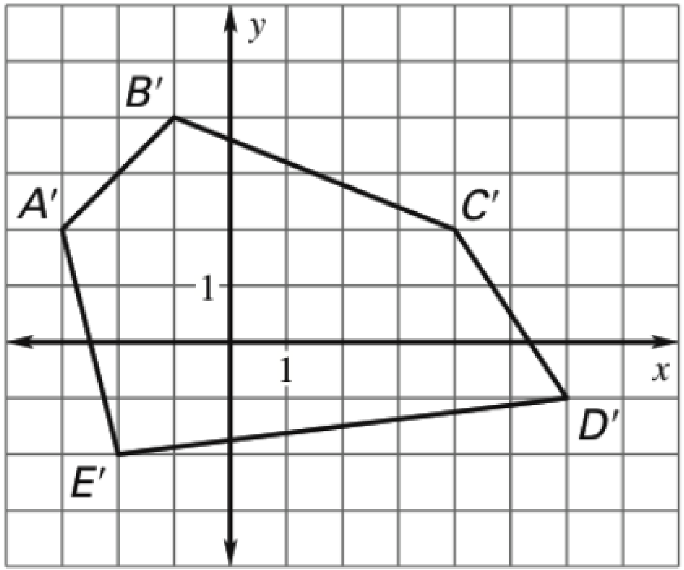
18. $\begin{bmatrix} 2 & 5 & 5 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ -4 \\ 2 \end{bmatrix}$

Use the described translation and the graph of the image to find the matrix that represents the preimage.

19. 3 units right and 4 units up



20. 2 units left and 3 units down



21. Matrix Equation Use the description of a translation of a triangle to find the value of each variable. What are the coordinates of the vertices of the image triangle?

$$\begin{bmatrix} -8 & x & -8 \\ 4 & 4 & y \end{bmatrix} + \begin{bmatrix} -2 & b & c \\ d & -5 & 2 \end{bmatrix} = \begin{bmatrix} r & -4 & -3 \\ 7 & s & 6 \end{bmatrix}$$

22. Office Supplies Two offices submit supply lists. A weekly planner costs $8, a chairmat costs $90, and a desk tray costs $5. Use matrix multiplication to find the total cost of supplies for each office.

Office 1	Office 2
15 weekly planners	25 weekly planners
5 chair mats	6 chair mats
20 desk trays	30 desk trays

23. School Play The school play was performed on three evenings. The attendance on each evening is shown in the table. Adult tickets sold for $5 and student tickets sold for $3.50.

Night	**Adults**	**Students**
First	340	250
Second	425	360
Third	440	390

a. Use matrix addition to find the total number of people that attended each night of the school play.

b. Use matrix multiplication to find how much money was collected from all tickets each night.