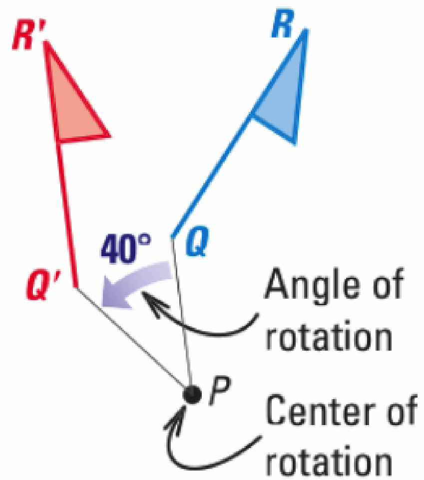
9.4 Perform Rotations

Recall from Lesson 4.8 that a *rotation* is a transformation in which a figure is turned about a fixed point called the **center of rotation**. Rays drawn from the center of rotation to a point and its image form the **angle of rotation**.

A rotation about a point P through an angle of $x°$ maps every point Q in the plane to a point Q' so that one of the following properties is true:

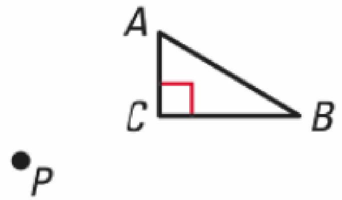
- If Q is not the center of rotation P, then $QP = Q'P$ and $m\angle QPQ' = x°$, or
- If Q is the center of rotation P, then the image of Q is Q.



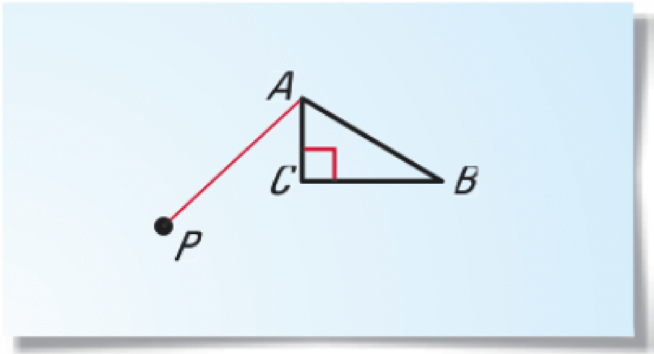
A 40° counterclockwise rotation is shown at the right. Rotations can be *clockwise* or *counterclockwise*. In this chapter, all rotations are counterclockwise.

EXAMPLE 1 Draw a rotation

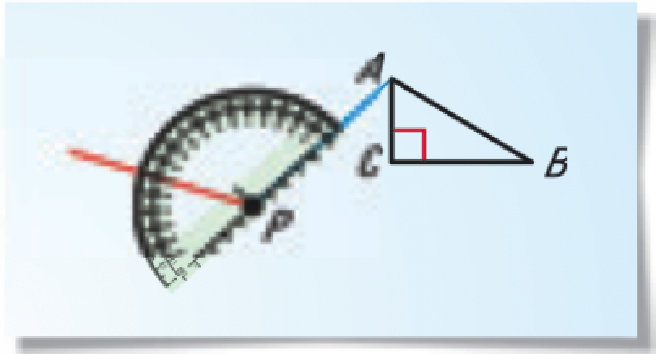
Draw a 120° rotation of $\triangle ABC$ about P.



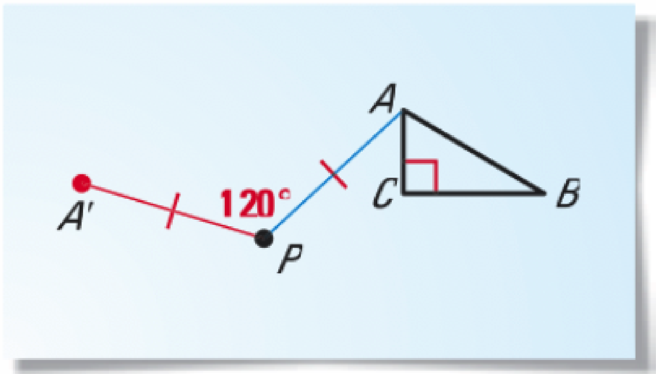
STEP 1 **Draw** a segment from A to P.



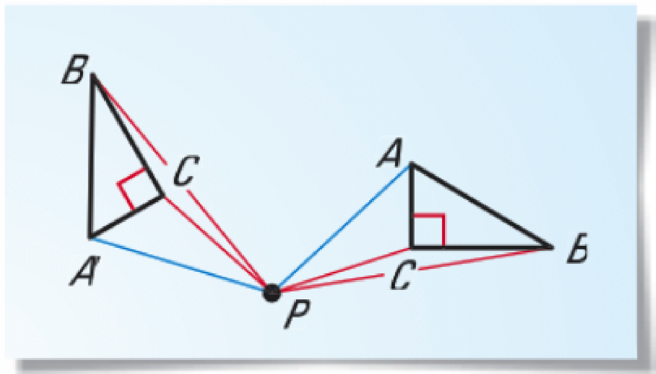
STEP 2 **Draw** a ray to form a 120° angle with $\overline{PA}$.



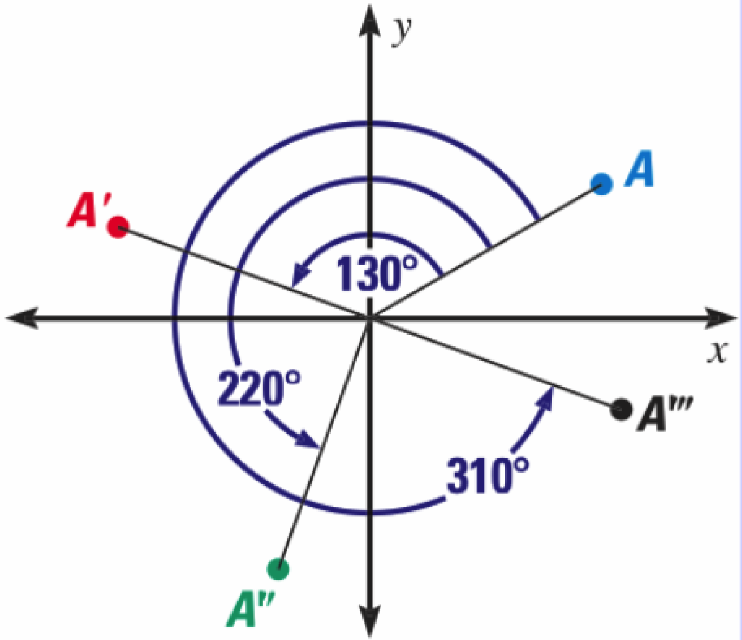
STEP 3 **Draw** A' so that $PA' = PA$.



STEP 4 **Repeat** Steps 1–3 for each vertex. Draw $\triangle A'B'C'$.



ROTATIONS ABOUT THE ORIGIN You can rotate a figure more than 180°. The diagram shows rotations of point *A* 130°, 220°, and 310° about the origin. A rotation of 360° returns a figure to its original coordinates.



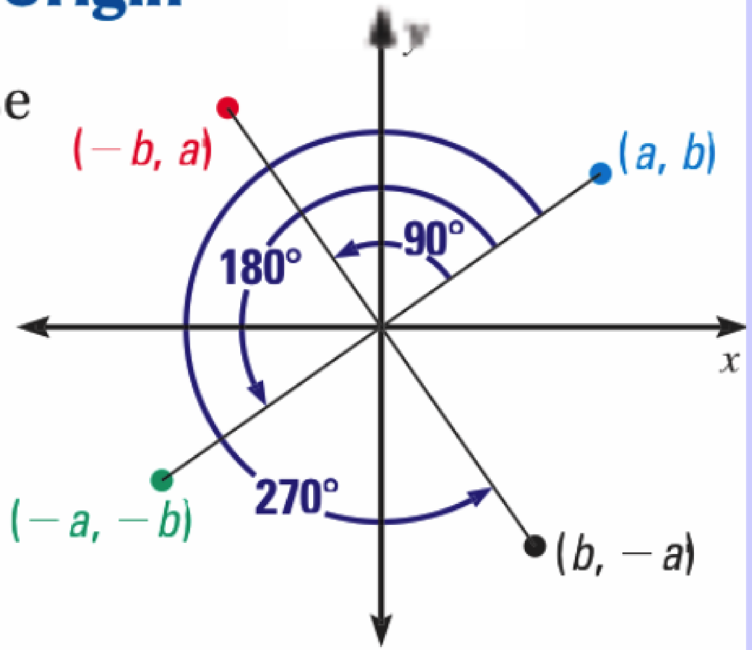
There are coordinate rules that can be used to find the coordinates of a point after rotations of 90°, 180°, or 270° about the origin.

KEY CONCEPT *For Your Notebook*

Coordinate Rules for Rotations about the Origin

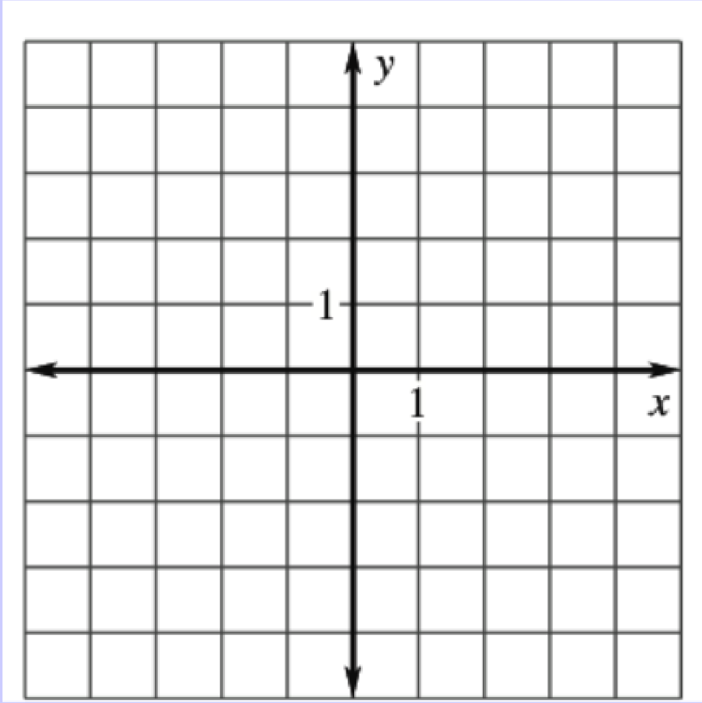
When a point (a, b) is rotated counterclockwise about the origin, the following are true:

1. For a rotation of 90°, $(a, b) \to (-b, a)$.
2. For a rotation of 180°, $(a, b) \to (-a, -b)$.
3. For a rotation of 270°, $(a, b) \to (b, -a)$.



EXAMPLE 2 Rotate a figure using the coordinate rules

Graph quadrilateral *RSTU* with vertices *R*(3, 1), *S*(5, 1), *T*(5, −3), and *U*(2, −1). Then rotate the quadrilateral 270° about the origin.



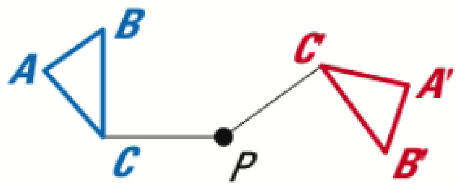
THEOREM

For Your Notebook

THEOREM 9.3 Rotation Theorem

A rotation is an isometry.

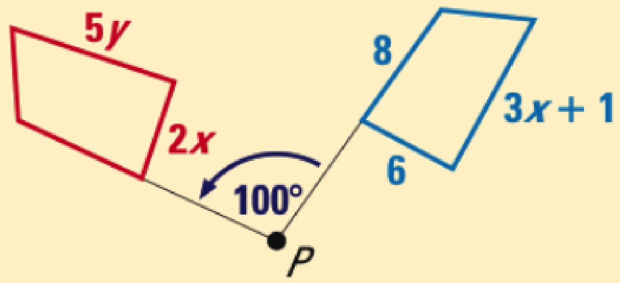
Proof: Exs. 33–35, p. 604



$\triangle ABC \cong \triangle A'B'C'$

EXAMPLE 4 Standardized Test Practice

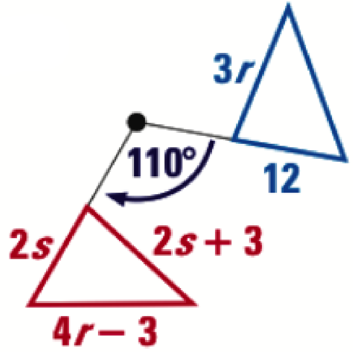
The quadrilateral is rotated about *P*. What is the value of *y*?



Ⓐ $\frac{8}{5}$ Ⓑ 2

Ⓒ 3 Ⓓ 10

6. Find the value of *r* in the rotation of the triangle.

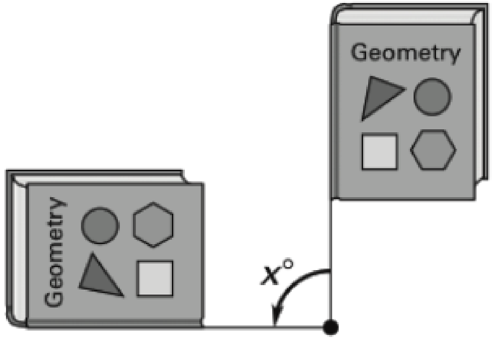


Ⓐ 3   Ⓑ 5

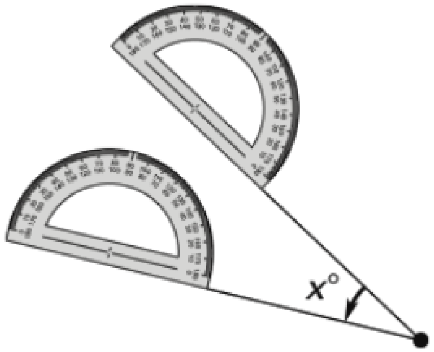
Ⓒ 6   Ⓓ 15

Match the diagram with the angle of rotation.

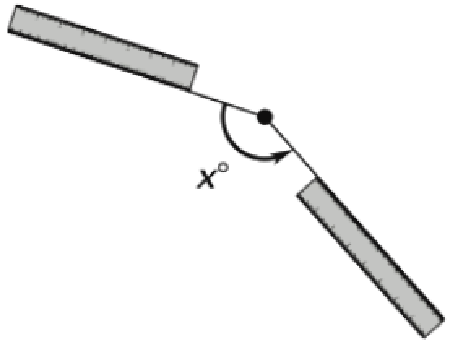
1.



2.



3.



A. 30°

B. 90°

C. 150°

Assignment:

9.4 ws

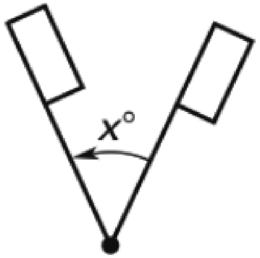
LESSON 9.4

Practice

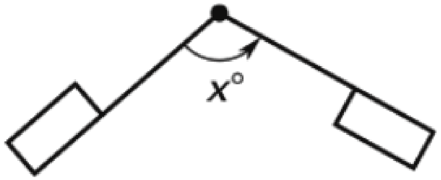
For use with pages 598–605

Match the diagram with the angle of rotation.

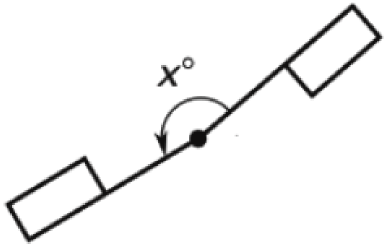
1.



2.



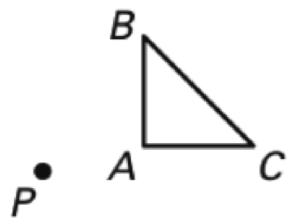
3.



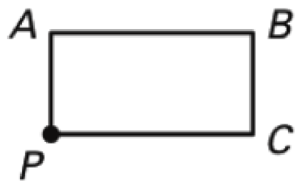
A. 110° **B.** 170° **C.** 50°

Trace the polygon and point *P* on paper. Then draw a rotation of the polygon the given number of degrees about *P*.

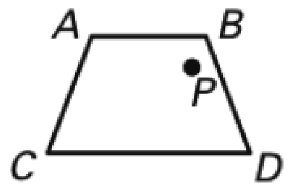
4. 45°



5. 120°

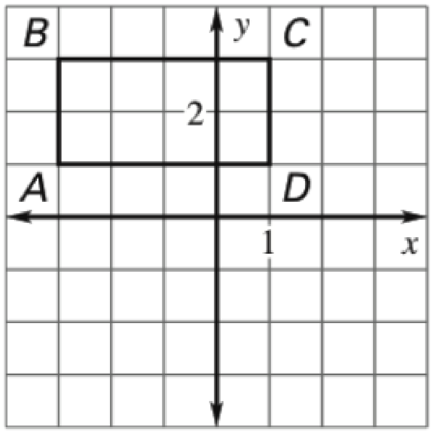


6. 135°

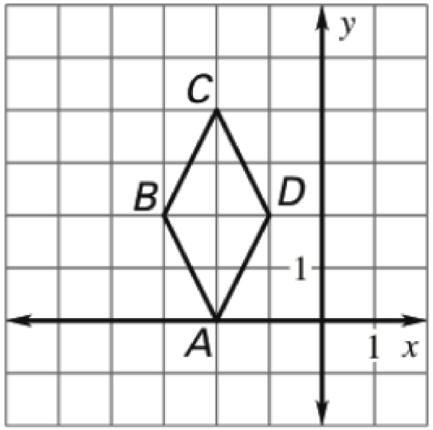


Rotate the figure the given number of degrees about the origin. List the coordinates of the vertices of the image.

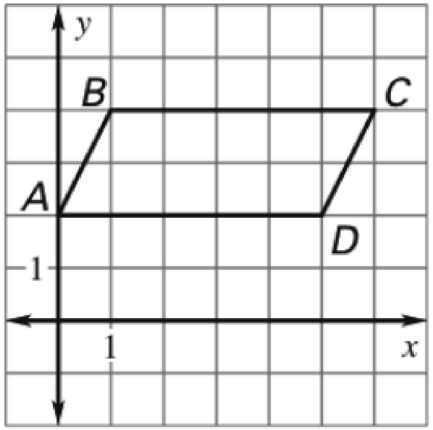
7. 90°



8. 180°

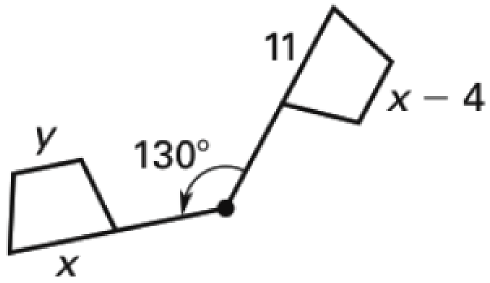


9. 270°

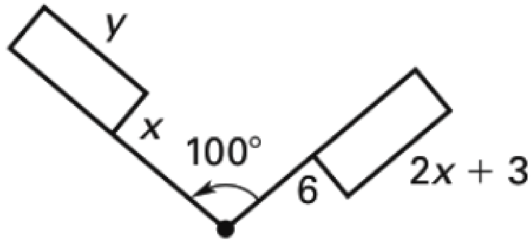


Find the value of each variable in the rotation.

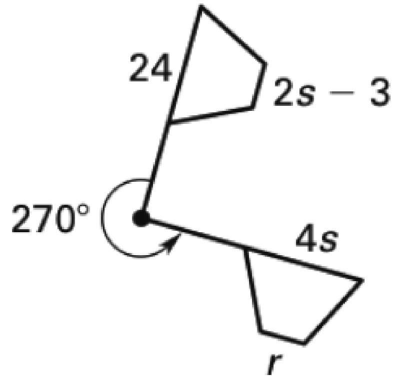
10.



11.

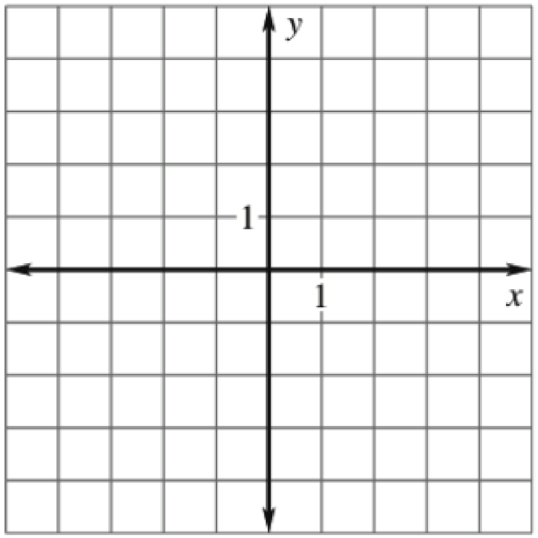


12.

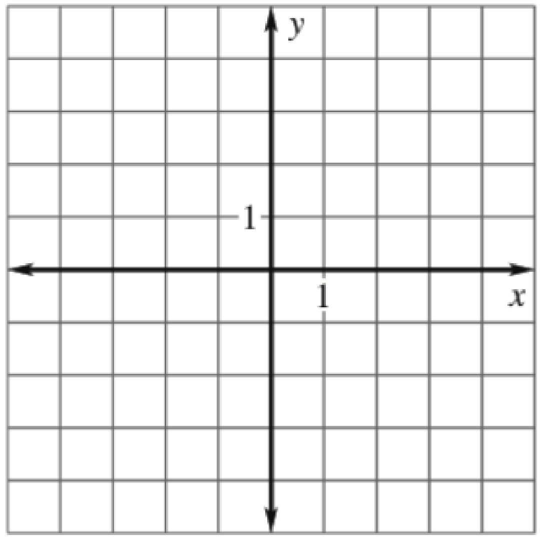


Find the image matrix that represents the rotation of the polygon about the origin. Then graph the polygon and its image.

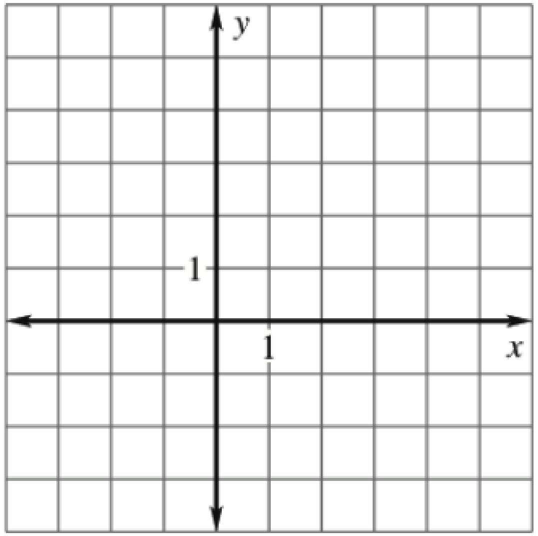
13. $\begin{matrix} A & B & C \\ \end{matrix}$ $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 2 & 4 \end{bmatrix}$; 90°



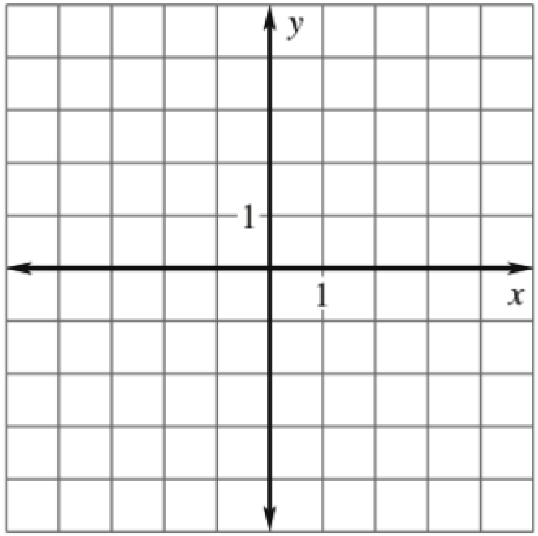
14. $\begin{matrix} A & B & C \\ \end{matrix}$ $\begin{bmatrix} 0 & 4 & 2 \\ -1 & 0 & 3 \end{bmatrix}$; 180°



15. $\begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 1 & 2 & 4 & 5 \\ -1 & 3 & 3 & -1 \end{bmatrix} \end{array}$; 90°

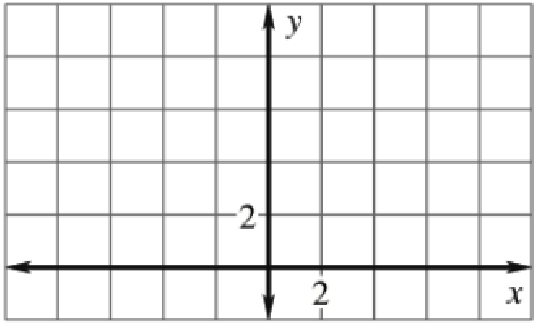


16. $\begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} -3 & -2 & 2 & 1 \\ -4 & -1 & -1 & -4 \end{bmatrix} \end{array}$; 270°



The endpoints of $\overline{CD}$ are $C(2, 1)$ and $D(4, 5)$. Graph $\overline{C'D'}$ and $\overline{C''D''}$ after the given rotations.

17. **Rotation:** 90° about the origin
Rotation: 270° about (2, 0)



18. **Rotation:** 180° about the origin
Rotation: 90° about (0, −3)